A Convex Optimization Approach to Robust Fundamental Matrix Estimation

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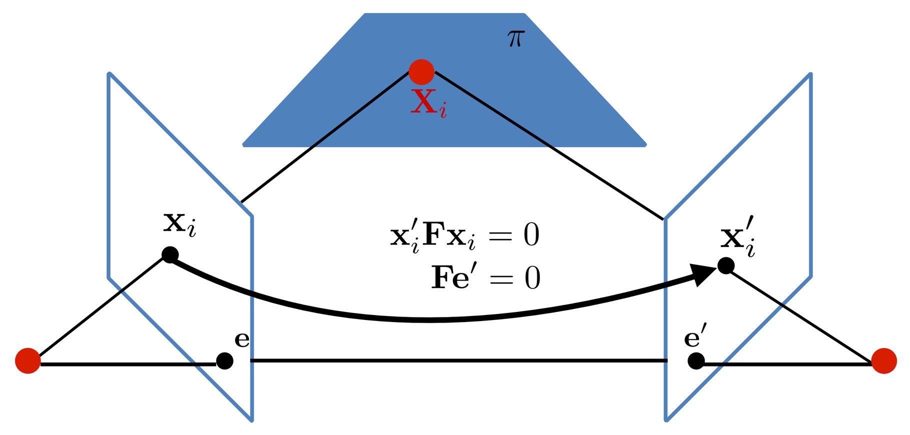
Motivation

Difficulties Estimating the Fundamental Matrix $\mathbf{F}$

- RANSAC and its variations (such as MSAC, LMEDS, MLESAC) are used to handle cases with outliers and noise. however, their performances deteriorate dramatically as the outliers increase.
- To enforce the **rank-2 constraint** on $\mathbf{F}$ is nontrivial.



Proposed QCQP Reformulation

Formulation 1:

$$p^* = \min_{\mathbf{v}} \; p(\mathbf{v}) \triangleq \sum_{i=1}^{n}(1-s_i) \qquad \cdots\cdots \quad \text{Number of Outliers}$$

$$\text{s.t.} \begin{cases} \|\mathbf{F}\|_F^2 = 1,\ \mathbf{q}^T\mathbf{q} = 1 & \cdots\cdots \quad \text{Normalization} \\ \mathbf{F}\mathbf{q} = 0 & \cdots\cdots \quad \mathbf{F}\text{: Rank-2, } \mathbf{q}\text{: Right epipole of } \mathbf{F} \\ s_i^2 = s_i, \forall_{i=1}^n & \cdots\cdots \quad s_i \in \{0,1\} \\ s_i|\mathbf{x}_i'^T\mathbf{F}\mathbf{x}_i| \le s_i\epsilon, \forall_{i=1}^n & \cdots\cdots \quad s_i = 1\text{: Inlier, } s_i = 0\text{: Outlier} \end{cases}$$

Variables: $\mathbf{v} = [\mathbf{q}^T \quad \mathbf{f}^T \quad s_1 \quad s_2 \quad \cdots \quad s_n]^T \in \mathbb{R}^{12+n}$, with $\mathbf{f} = \text{vec}(\mathbf{F})$

Standard SDP Based Relaxation

QCQP

$$p^* = \min_{\mathbf{v}\in\mathbb{R}^D} \mathbf{v}^T\mathbf{Q}_0\mathbf{v} + 2\boldsymbol{\ell}_0^T\mathbf{v} + r_0$$
$$\text{s.t. } \mathbf{v}^T\mathbf{Q}_i\mathbf{v} + 2\boldsymbol{\ell}_i^T\mathbf{v} + r_i \le 0,\ \forall_{i=1}^d$$

Convex if and only if $\mathbf{Q}_i \succeq 0,\ \forall_{i=0}^d$

Replace $\mathbf{V} = \begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}[1 \quad \mathbf{v}^T]$ **by**

$$\mathbf{M}_1 \triangleq \begin{bmatrix} m(1) & m(v_1) & \cdots & m(v_D) \\ m(v_1) & m(v_1^2) & \cdots & m(v_1 v_D) \\ \vdots & \vdots & \ddots & \vdots \\ m(v_D) & m(v_D v_1) & \cdots & m(v_D^2) \end{bmatrix}$$

$m(\bullet)$ denotes the entry of $\mathbf{M}_1$ corresponding to $\bullet$ in $\mathbf{V}$

SDP-Based Relaxation

$$\tilde{p}^* = \min_{\mathbf{M}_1\in\mathbb{R}^{(D+1)\times(D+1)}} \text{Trace}(\tilde{\mathbf{Q}}_0\mathbf{M}_1)$$
$$\text{s.t.} \begin{cases} \text{Trace}(\tilde{\mathbf{Q}}_i\mathbf{M}_1) \le 0,\ \forall_{i=1}^d \\ \mathbf{M}_1(1,1) = 1 \\ \mathbf{M}_1 \succeq 0 \end{cases}$$

with $\tilde{\mathbf{Q}}_i = \begin{bmatrix} r_i & \boldsymbol{\ell}_i^T \\ \boldsymbol{\ell}_i & \mathbf{Q}_i \end{bmatrix}, \forall_{i=0}^d$

$\tilde{p}^* \le p^*$.
When $\text{Rank}(\mathbf{M}_1) = 1$ $\tilde{p}^* = p^*$,
and $\mathbf{v}^* = m(\mathbf{v})^*$ is the global optimizer.

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Reduced SDP Based Reformulation

Directly applying SDP-relaxation would introduce a SDP constraint $\mathbf{M}_1 \succeq 0$ and $\mathbf{M}_1$ is of size $(14+n)\times(14+n)$.

Take $n = 2$ as an example:

common variables $\mathbf{M}_1^c \in \mathbb{R}^{14\times 14}$ $\quad (\mathbf{x}_1, \mathbf{x}_1') \quad (\mathbf{x}_2, \mathbf{x}_2')$

$$\mathbf{M}_1 = \begin{bmatrix} m(1) & m(\mathbf{f})^T & m(\mathbf{q})^T & m(s_1) & m(s_2) \\ m(\mathbf{f}) & m(\mathbf{f}\mathbf{f}^T) & m(\mathbf{f}\mathbf{q}^T) & m(\mathbf{f}s_1) & m(\mathbf{f}s_2) \\ m(\mathbf{q}) & m(\mathbf{q}\mathbf{f}^T) & m(\mathbf{q}\mathbf{q}^T) & m(\mathbf{q}s_1) & m(\mathbf{q}s_2) \\ m(s_1) & m(s_1\mathbf{f}^T) & m(s_1\mathbf{q}^T) & m(s_1^2) & m(s_1 s_2) \\ m(s_2) & m(s_2\mathbf{f}^T) & m(s_2\mathbf{q}^T) & m(s_2 s_1) & m(s_2^2) \end{bmatrix}$$

$$\mathbf{M}_{1,1} = \begin{bmatrix} m(1) & m(\mathbf{f})^T & m(\mathbf{q})^T & m(s_1) \\ m(\mathbf{f}) & m(\mathbf{f}\mathbf{f}^T) & m(\mathbf{f}\mathbf{q}^T) & m(\mathbf{f}s_1) \\ m(\mathbf{q}) & m(\mathbf{q}\mathbf{f}^T) & m(\mathbf{q}\mathbf{q}^T) & m(\mathbf{q}s_1) \\ m(s_1) & m(s_1\mathbf{f}^T) & m(s_1\mathbf{q}^T) & m(s_1^2) \end{bmatrix}$$

$$\mathbf{M}_{2,1} = \begin{bmatrix} m(1) & m(\mathbf{f})^T & m(\mathbf{q})^T & m(s_2) \\ m(\mathbf{f}) & m(\mathbf{f}\mathbf{f}^T) & m(\mathbf{f}\mathbf{q}^T) & m(\mathbf{f}s_2) \\ m(\mathbf{q}) & m(\mathbf{q}\mathbf{f}^T) & m(\mathbf{q}\mathbf{q}^T) & m(\mathbf{q}s_2) \\ m(s_2) & m(s_2\mathbf{f}^T) & m(s_2\mathbf{q}^T) & m(s_2^2) \end{bmatrix}$$

Formulation 2: (Linear+Reduced SDP)

$$\tilde{p}^* = \min_{\{\mathbf{M}_{i,1}\}_{i=1}^n} \sum_{i=1}^{n}[1 - m(s_i)]$$

$$\text{s.t.} \begin{cases} m(\|\mathbf{F}\|_F^2) = 1,\ m(\mathbf{q}^T\mathbf{q}) = 1 \\ m(\mathbf{F}\mathbf{q}) = 0 \\ m(s_i^2) = m(s_i), \forall_{i=1}^n \\ |\mathbf{x}_i'^T m(s_i\mathbf{F})\mathbf{x}_i| \le m(s_i)\epsilon, \forall_{i=1}^n \\ \mathbf{M}_1^c(1,1) = 1 \\ \mathbf{M}_{i,1} \succeq 0, \forall_{i=1}^n \end{cases}$$

$\cdots\cdots$ Linear $\mathbf{L} \le 0$

$\cdots\cdots$ Reduced SDP

Variables: $\mathbf{M}_{1,1}, \mathbf{M}_{2,1}, \cdots, \mathbf{M}_{n,1} \in \mathbb{R}^{15\times 15}$

When $\text{Rank}(\mathbf{M}_1^c) = 1$ holds, $\mathbf{M}_{i,1} \succeq 0,\ \forall_{i=1}^n$ guarantees the existence of $\mathbf{M}_1$ such that $\mathbf{M}_1 \succeq 0,\ \text{Rank}(\mathbf{M}_1) = 1$. Thus, $\tilde{p}^* = p^*$, and $\mathbf{v}^* = m(\mathbf{v})^*$ is the global optimizer.

- **Formulation: explicitly enforces the rank-2 constraint on $\mathbf{F}$;**
  s_i provides the flexibility of exploiting a-priori information, i.e. co-occurrences;
- **Theoretically gives sufficient conditions guaranteeing the global optimality of the SDP relaxation;**
- **Practically improves the performance in the presence of a large percentage of outliers.**

Proposed Iterative Algorithm

Formulation 3: (Linear+Reduced SDP+Re-weighted Nuclear Norm)

Initialize: $k = 0,\ \mathbf{W}^{(1)} = \mathbf{I} \in \mathbb{R}^{14\times 14}$

Repeat

$k = k + 1;$

obtain $\mathbf{M}_1^{c*(k)}$ by solving

$$\tilde{p}^{(k)*} = \min_{\{\mathbf{M}_{i,1}\}_{i=1}^n} \sum_{i=1}^{n}[1 - m(s_i)] + \lambda\text{Trace}(\mathbf{W}^{(k)}\mathbf{M}_1^c)$$
$$\text{s.t.} \quad \mathbf{L} \le 0,\ \mathbf{M}_{i,1} \succeq 0, \forall_{i=1}^n$$

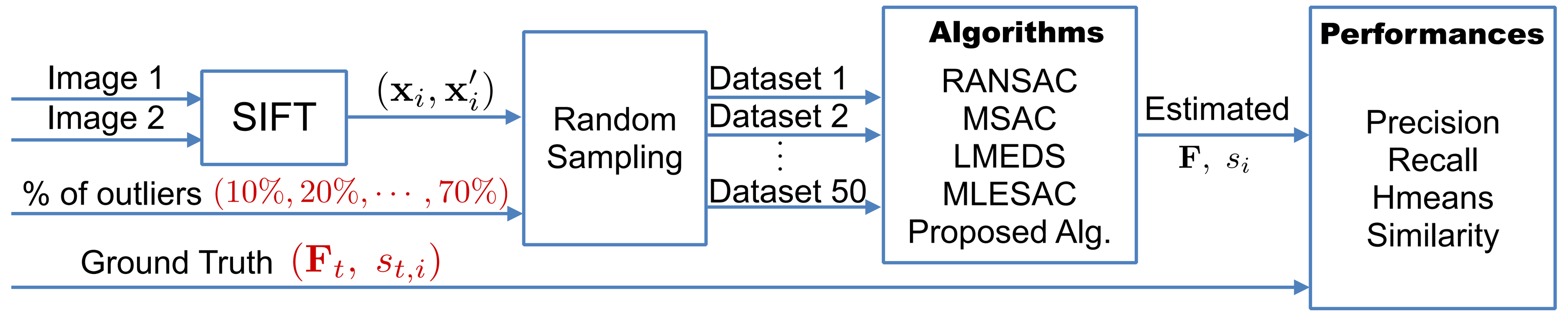
update $\quad \mathbf{W}^{(k+1)} = [\mathbf{M}_1^{c*(k)} + \sigma_2(\mathbf{M}_1^{c*(k)})\mathbf{I}]^{-1};$

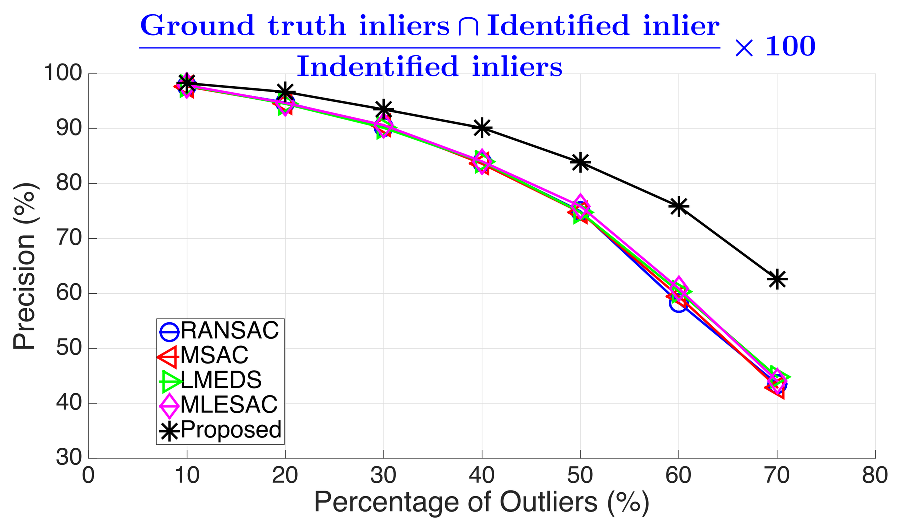
Until $\sigma_2(\mathbf{M}_1^{c*(k)}) \le 10^{-6}\sigma_1(\mathbf{M}_1^{c*(k)})$ **, (numerically converges to Rank-1 solution).**

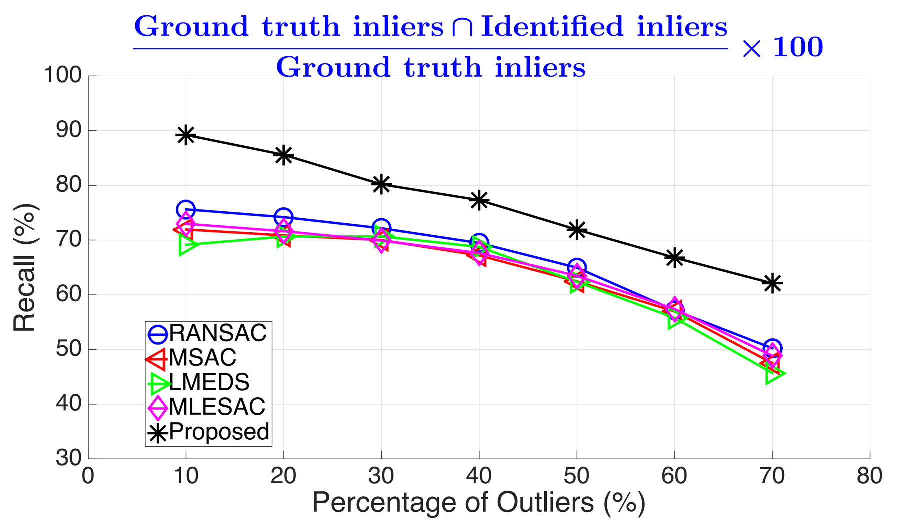
(Source code: http://robustsystems.coe.neu.edu/ or scan the QR code above)

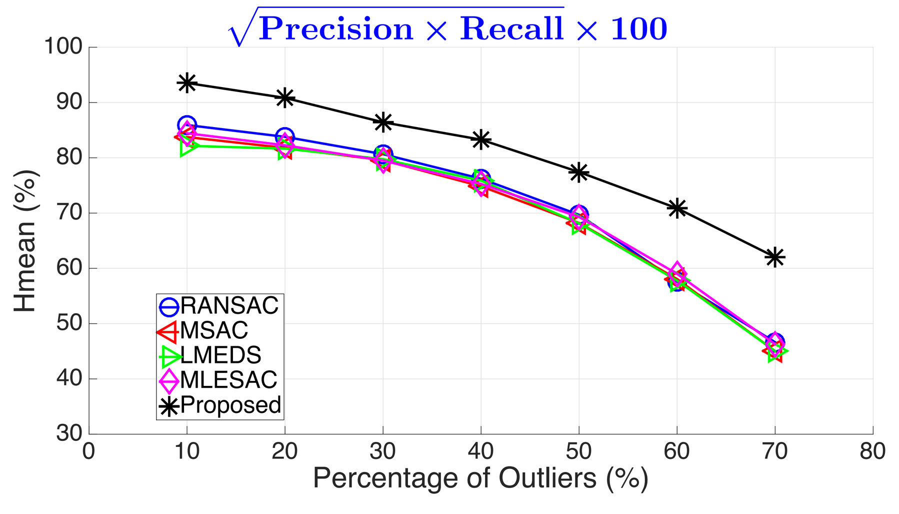
Experiments

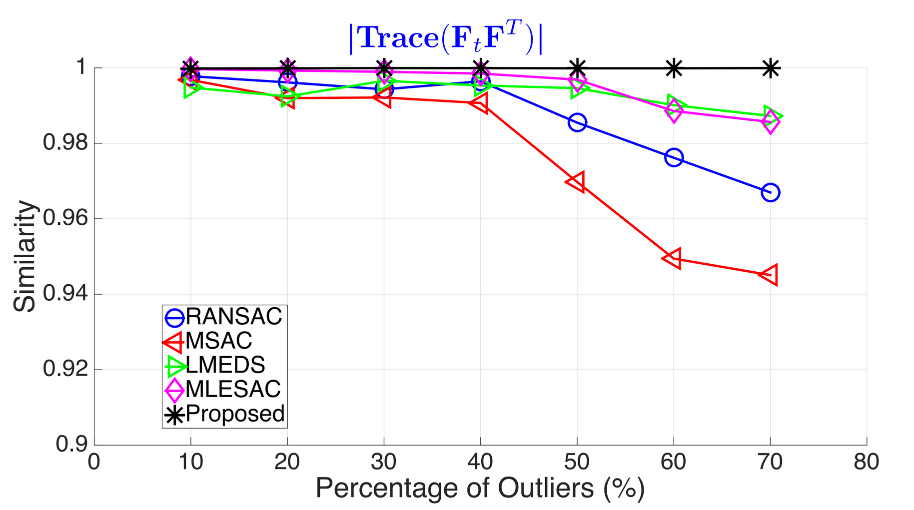
Estimate fundamental matrices of six pairs of stereo images: **{House, Merton I, Merton II, Merton III, Library, Wadham}**, given by VGG, University of Oxford.











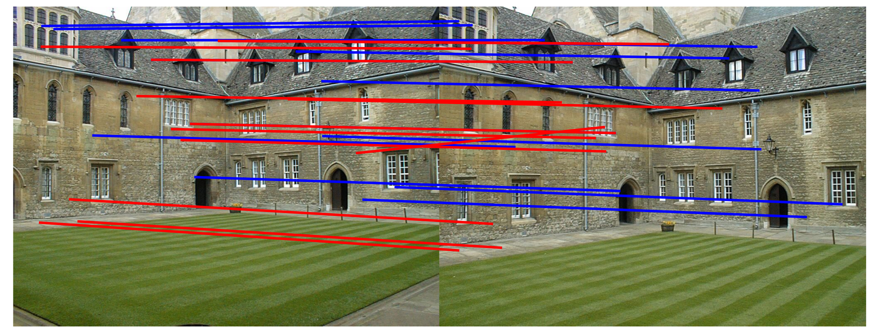
Sample Images of Merton I

Ground Truth Inliers (30 pairs)

Ground Truth Outliers (70 pairs)

Blue: False Outliers, Red: False Inliers

Best among the Cited Methods (LMEDS)



Proposed Algorithm