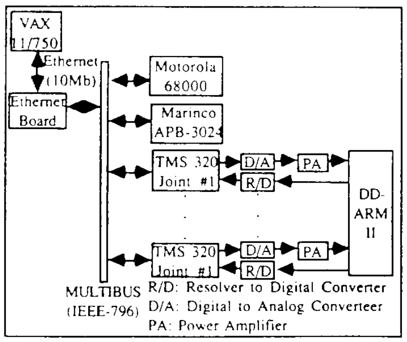

Fig. 22. Controller architecture of CMU DD Arm II.

achieved a computation time of 1 ms by implementing these on the Marinco processor. The details of the customized algorithm, hardware configuration, and the numerical values of the dynamics parameters are presented in [6].

References

[1] C. H. An, C. G. Atkeson, and J. M. Hollerbach, "Experimental determination of the effect of feedforward control of trajectory tracking errors," in A. K. Bejczy, Ed., *Proc. 1986 IEEE Conf. on Robotics and Automation* (IEEE, San Francisco, CA, Apr. 7-10, 1986), pp. 55-60.

[2] H. Asada and T. Kanade, "Design of direct drive mechanical arms," *J. Vibration, Stress, Reliab. Des.*, vol. 105, no. 1, pp. 312-316, July 1983.

[3] H. Asada, T. Kanade, and I. Takeyama, "Control of a direct-drive arm," Tech. Rep. CMU-RI-TR-82-4, The Robotics Institute, Carnegie-Mellon Univ., Apr. 1982.

[4] J. M. Hollerbach, "Dynamic scaling on manipulator trajectories," *J. Dyn. Sys., Meas., Contr.*, vol. 106, no. 1, pp. 102-106, Mar. 1984.

[5] T. Kanade, P. K. Khosla, and N. Tanaka, "Real-time control of the CMU Direct Drive Arm II using customized inverse dynamics," in M. P. Polis, Ed., *Proc. 23rd IEEE Conf. Decision and Control* (Las Vegas, NV, Dec. 12-14, 1984), pp. 1345-1352.

[6] P. K. Khosla, "Real-time control and identification of direct-drive manipulators," Ph.D. dissertation, Dep. Elec. Comput. Eng., Carnegie-Mellon Univ., Aug. 1986.

[7] P. K. Khosla and T. Kanade, "Parameter identification of robot dynamics," in G. F. Franklin, Ed., *Proc. 24-th CDC* (Florida, Dec. 11-13, 1985), pp. 1754-1760.

[8] —, "Real-time implementation and evaluation of model-based controls on CMU DD ARM II," in A. K. Bejczy, Ed., *1986 IEEE Int. Conf. on Robotics and Automation* (IEEE, Apr. 7-10, 1986).

[9] M. B. Leahy, K. P. Valavanis, and G. N. Saridis, "The effects of dynamics models on robot control," in *Proc. 1986 IEEE Conf. on Robotics and Automation* (IEEE, San Francisco, CA, Apr. 1986). See also this issue pp. 000-000.

[10] J. Y. S. Luh, M. W. Walker, and R. P. Paul, "Resolved-acceleration control of mechanical manipulators," *IEEE Trans. Automat. Contr.*, vol. AC-25, no. 3, pp. 468-474, June 1980.

[11] Marinco APB-3024M Array Processor Board, *Reference Manual.* San Diego, CA: Marinco, Inc. (3878-A Ruffin Road, San Diego, CA 92123), 1983.

[12] B. R. Markiewicz, "Analysis of the computed-torque drive method and comparison with the conventional position servo for a computer-controlled manipulator," Tech. Memo. 33-601, Jet Propulsion Lab., Pasadena, CA, Mar. 1973.

[13] R. P. Paul, *Robot Manipulators: Mathematics, Programming and Control.* Cambridge, MA: MIT Press, 1981.

[14] D. Schmitz, P. K. Khosla, and T. Kanade, "Development of CMU Direct-Drive Arm II," in Y. Hasegawa, Ed., *Proc. 15-th Int. Symp. on Industrial Robotics* (Tokyo, Japan, Sept. 11-13, 1985).

An Adaptive Controller for a One-Legged Mobile Robot

M. SZNAIER AND M. J. DAMBORG

Abstract—**An adaptive controller based upon the on-line minimization of a performance criteria is described. The adaptive controller is used to improve the performance of a one-legged mobile robot, removing problems experienced with previous controllers. The performance of several minimization algorithms is analyzed and, as a result, the Adaptive Step Size Random Search algorithm is selected. Finally, a series of experiments illustrating the ability of the adaptive controller to handle a changing environment is presented.**

I. Introduction

Walking machines pose two broad categories of problems: those related to movement over uneven terrain and those related to dynamic balance while walking and running.

The first class of problems can be explored with machines that are inherently stable. The need for dynamic balance is avoided by having enough legs [1], [2]. Several six-legged, computer-controlled machines have been built that perform satisfactorily. Their movement resembles the crawling displayed by insects, achieving a velocity of about 2 mi/h [3].

The second class of problems, namely, those related to dynamic balance, are the typical problems that arise in the study of animal and bipedal locomotion. In contrast to a crawling machine, a dynamically balanced vehicle can be allowed to tip for brief intervals as long as an adequate base of support is provided on the average. Dynamically stabilized legged systems have various modes of motion. They can either walk, in which case some of the legs provide support while the others move, or they can run and jump, in which case there are time intervals where all the legs leave the ground. A comprehensive discussion of dynamically stabilized walking machines and their applications can be found in [4].

The simplest dynamically stabilized system is a unipedal system, which can move only by jumping. The importance of this uniped resides in the fact that it provides a good model for studing the relevant problems involved in the walking and running motions of dynamically stabilized legged machines without the complexity associated with systems having a higher number of legs. Although the motion of legged machines takes place in a three-dimensional space, the control of such systems along a straight path does not need to deal explicitly with three-dimensional dynamics. It is possible to decompose the control law to a planar and an extraplanar part where the task of this extraplanar part is to restrict the motion to a plane. Most of the motion will take place then in a plane with small extraplanar movements. In this case, a simple two-dimensional model will be enough to study the problem and to design an effective control system that can be ported to the actual robot [5]. Raibert [6]-[8] designed a controller that allows a bidimensional, one-legged hopping robot to move from one point to another with a given velocity. The resulting algorithms were successfully ported to a physical, computer-controller machine. However, as is reported in [8], the application of this controller results in a steady-state velocity error or bias that depends on the forward velocity and on the parameters of the model.

In this communication we propose a new control law for the vertical controller that reduces the coupling between the horizontal and

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vertical motions. We also propose an adaptive control law for the horizontal controller which not only completely removes the bias in the forward velocity, but does so adaptively to off-nominal conditions. Through simulations it is shown that the adaptive control law can also compensate for different velocities and for deviations from the nominal conditions that were used for the controller design. These deviations are intended to model conditions that a robot might be subjected to such as changing mass due to different payloads or changes in the environmental characteristics such as ground softness or slope. Our adaptive controller proves capable of handling such variations thereby providing a dynamically stabilized legged robot with enough flexibility to function in a more realistic environment.

II. Description of the Problem

A. Statement of the Problem

In [7], Raibert presents a control algorithm that allows a dynamically stabilized, one-legged robot to travel from one place to another at a prescribed velocity while maintaining a predetermined hopping height. In this communication our goal is to design a control algorithm that will remove the steady-state velocity error experienced by Raibert and will allow the robot to sustain a stable motion with a given velocity under widely variable conditions.

Throughout this communication we will employ the model developed in [7]. This model describes a computer-controlled "pogo stick," consisting of a body (which will carry all the electronics and any payload) and a springy, nonzero mass leg that articulates with respect to the body. The leg is modeled as a spring attached on one end to a foot and on the other to a position actuator that is used to control its overall length and to inject energy into the system. The control inputs to the system are a torque T applied at the hip and the displacement X of the position actuator. Finally, the ground is modeled as a spring K_G and damper B_G, in both the vertical and horizontal direction. The device is illustrated in Fig. 1.

B. Survey of Raibert's Control Algorithms

Raibert [7] separates the controller design into a vertical and a horizontal controller that are assumed to be decoupled:

Vertical Controller: The tasks of the vertical motion controller are to initiate and terminate the hopping and to control its height. All of these tasks are accomplished by changing the energy of the resonant mass–spring system formed by the leg and the body using changes in the length of the position actuator.

Horizontal Controller: The task of the horizontal controller is twofold: it must control the forward velocity and it must maintain reasonable values for the body attitude to prevent the robot from tipping.

Hopping systems are able to change their linear and angular momentum only during stance since it is only during this phase that external forces act on the system. The reaction of the ground generates a torque about the center of gravity which is proportional to the displacement of the foot and that can be used to modify the angular and horizontal accelerations. Consider the trajectory described by the center of gravity of the system during stance; the horizontal projection of this trajectory is called the "CG print." Assuming that the motion is perfectly symmetric around the center of the CG print, then, if the foot is placed exactly there, the reaction of the ground generates no net torque around the center of mass. Applying the same symmetry considerations, we have that the net horizontal force is zero and thus the system maintains its present velocity. This point in the CG print that generates no net acceleration is called the neutral point. If the foot is displaced from the neutral point, a net torque that modifies the horizontal velocity is generated. To control the forward velocity, Raibert [7] uses a linear law to determine the displacement of the foot relative to the neutral point

$$x_{\text{err}} = K_x(\dot{x}_{\text{CG}} - \dot{x}_{\text{ref}}) \tag{1}$$

where $\dot{x}_{\text{CG}}$ is the velocity of the center of gravity during the last

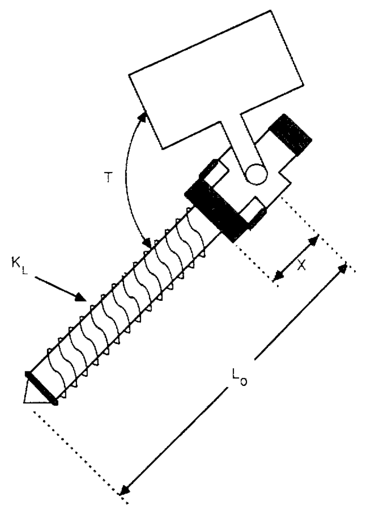


Fig. 1. The physical model.

flight, $\dot{x}_{\text{ref}}$ is the desired forward velocity, and K_x is a constant determined experimentally. The landing position of the foot is computed as the estimated position of the neutral point plus the displacement x_{err} and the leg is driven to the desired position by applying a torque a the hip.

The second task of the horizontal controller is to prevent the robot from tipping by maintaining the body attitude angle within certain limits. This task can be accomplished by simply driving the body attitude to zero during stance.

C. Analysis of the Simulation of the Control Algorithms

Fig. 2 shows the results of the simulation of Raibert's control law. After a transient that decays quickly, the robot reaches a steady state characterized by the fact that the velocity remains roughly constant from hop to hop. The simulations show the existence of a velocity-dependent steady-state velocity error. The existence of this error means that, under steady-state conditions, the position of the neutral point is displaced from the center of the CG print by an amount given by (1).

To examine the origin of the displacement of the neutral point, we need a more thorough analysis of the robot's motion during stance. As a first approximation to this motion, we will study the motion of the simple mass–spring system shown in Fig. 3. The system consists of a mass m positioned on top of a spring with rest length l_0 and constant k. A position actuator of length X acts between them and will serve as our control input. For simplicity, this system will be restricted to unidimensional motion. Our goal is to explore the force exerted by the ground on the system and to find a control input $X(t)$ that will cause this force to be symmetric (in time) around the lowest point of the trajectory.

The Newton equation applied to the system in the y direction is

$$m\ddot{y} = -mg - k(y - X(t) - l_0). \tag{2}$$

The initial conditions for the system are assumed to be: $y(0) = l_0$ (rest length), $\dot{y}(0) = -v_0$ (downward velocity), and the numerical values for the coefficients: $m = 10$ kg, $k = 1000$ N/m, $l_0 = 1$ m, $v_0 = 5$ m/s. If we assume a piecewise-linear input $X(t)$

$$X(t) = \begin{cases} 0, & \text{for } 0 < t < t_1 \\ a(t - t_1), & \text{for } t_1 < t < t_2 \\ a(t_2 - t_1), & \text{for } t_2 < t \end{cases} \tag{3}$$

where t_1 and t_2 are parameters to be determined. Then, neglecting

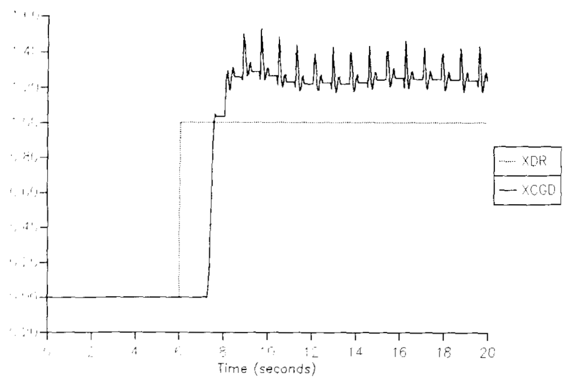


Fig. 2. Velocity response of Raibert's algorithm for $K_x = 0.09$ and reference velocity = 1 m/s.

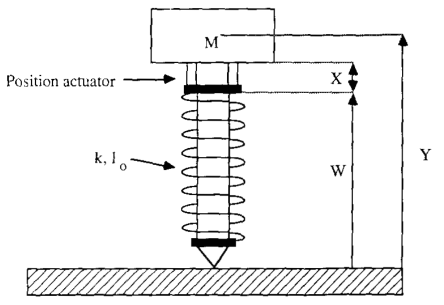


Fig. 3. A simple model to study motion during stance.

the term mg in (2), the solution for the force $F(t)$ is given by

$$\begin{aligned} F(t) &= -k(y(t) - X(t) - l_0) = v_0\sqrt{mk}\sin w_m t, \qquad t < t_1 \\ &= v_0\sqrt{mk}\sin w_m t + a\sqrt{mk}\sin w_m(t - t_1), \\ &\qquad\qquad t_1 < t < t_2 \\ &= v_0\sqrt{mk}\sin w_m t + a\sqrt{mk}\sin w_m(t - t_1) \\ &\quad -a\sqrt{mk}\sin w_m(t - t_2), \qquad t_2 < t \qquad (4) \end{aligned}$$

where

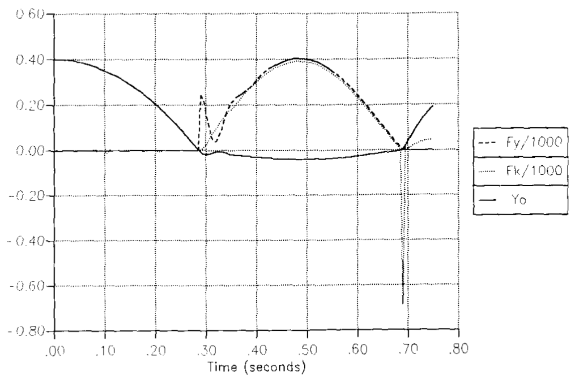
$$w_m = \sqrt{k/m}.$$

From these expressions it is clear that, in order to get a symmetric force around the lowest point of the trajectory, we must hve

$$t_1 = 0 \qquad t_2 = \pi\sqrt{m/k} \qquad (5)$$

that is, the leg should be expanded linearly (in time) starting at touchdown and finishing at liftoff in order to cancel the effects of the vertical motion controller upon the horizontal motion and conserve the overall symmetry required by Raibert's algorithm. A simulation of the full two-dimensional robot hopping in place, with the linear expansion of the leg beginning at touchdown, is shown in Fig. 4(a). Here, F_y is the vertical force acting on the foot and F_k is the total force exerted by the spring. The plots show that the results agree with the analysis and that the forces are sinusoidal except during a brief interval at touchdown and liftoff where collision phenomena occur.

The situation departs from the ideal case analyzed when the robot is running as is shown in Fig. 4(b). Even though the basic analysis of the vertical motion remains correct there is now a nonsymmetric horizontal force (F_x). During stance, this force accelerates the robot



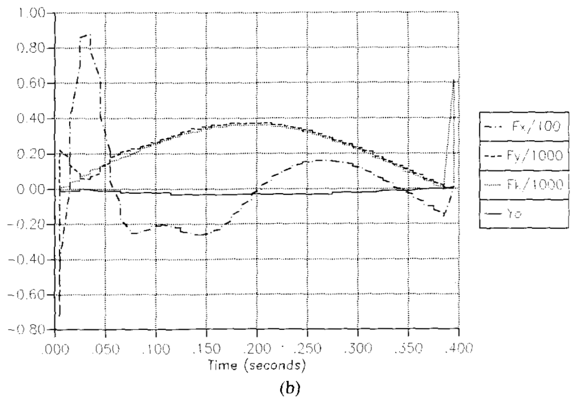


Fig. 4. (a) Forces acting on the robot while hopping in place. (b) Forces acting on the robot while running.

resulting in a larger than expected velocity during part of the trajectory. This, in turn, results in a larger than estimated CG print. The consistent underestimation of the CG print coupled to the asymmetric nature of the force generates a forward displacement of the neutral point. Hence, to maintain its present velocity, the robot must displace the foot from the center of the CG print by means of a positive velocity error.

III. Adaptive Control Algorithm

A. Modification of the Control Algorithm

In the last section we identified the displacement of the neutral point from the center of the CG print as the source of the steady-state error in velocity. This displacement can be counteracted by modifying (1) which computes the displacement of the foot to

$$x_{\text{err}} = K_x(\dot{x}_{\text{CG}} - \dot{x}_{\text{ref}}) + K_{\text{corr}} \qquad (6)$$

where the term K_{corr} provides the required correction to the displacement. In principle, K_{corr} is a function of the velocity and of the parameters of the model (such as mass and the constants of the ground model). However, because of the complex dynamics of the system, the form of this function is not known. Rather than deriving or estimating a relationship between K_{corr} and the velocity error we will use an idea central to adaptive control. We will couple the original control algorithm to an on-line numerical minimization procedure that will attempt to minimize the square of the velocity error, considered as a function of the correction term. The square of the velocity error as a function of K_{corr} in the relevant range is shown in Fig. 5. From the observed displacement of the neutral point in simulations this relevant range is determined to be the interval [0.007 0.025]. The plot shows that the function is well-behaved and almost quadratic in this interval, suggesting that the numerical minimization will converge rapidly.

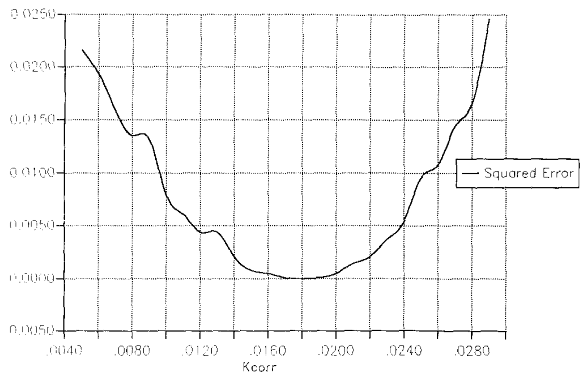
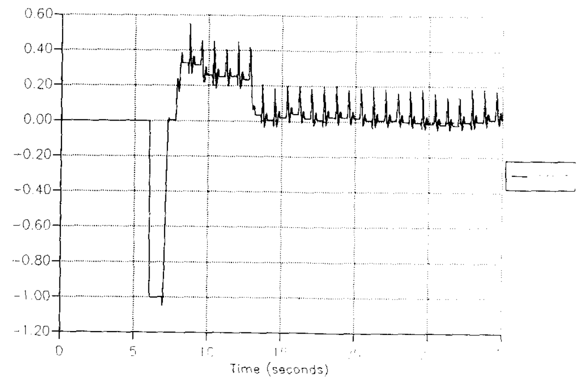
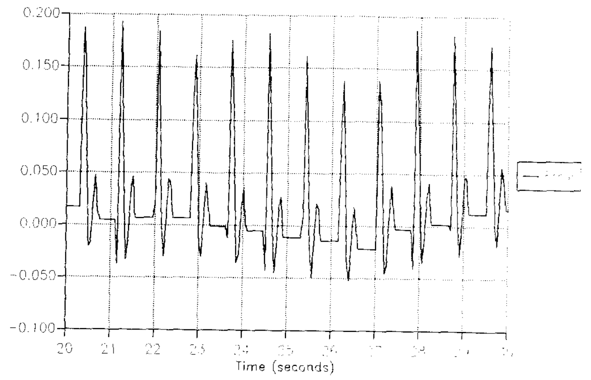


Fig. 5. The error^2 as a function of K_{corr}.



(a)



(b)

Fig. 6. (a) Velocity error for the ASSRS algorithm. (b) Velocity error for the ASSRS algorithm.

B. Proposed Adaptive Horizontal Control Algorithm [10]

In Section II-C we saw that the response to a step in the reference velocity is characterized by a transient period, in which the velocity changes rapidly, followed by a steady-state regime in which it remains roughly constant from hop to hop. Hence, the adaptive control algorithm is composed of two parts:

1) The original algorithm proposed by Raibert, modified by replacing (1) for computing the displacement of the foot by (6).

2) An on-line minimization procedure for minimizing the square of the velocity error which is considered to be a function of the correction term K_{corr} in (6).

Part 1 of the algorithm is responsible for controlling the robot during the the transient. Part 2, the adaptive part, will function during steady-state operation and its purpose is to zero the steady-state velocity error generated by part 1. During the transient period the value of the correction term is kept constant.

We will consider that a steady-state situation is reached when the absolute and relative changes in error from one hop to the next are both less than a threshold which is a design parameter. This threshold represents a compromise between robustness and speed of response of the algorithm. Once the steady-state situation is reached, the on-line minimization is activated. If, at some point, both the absolute and relative changes in the error exceed the threshold, the controller assumes that a change in some of the parameters (mass, ground resiliency, slope) has occurred and the algorithm reverts to part 1 until a steady-state situation is reached again.

C. Analysis of the Different Alternatives for the On-Line Minimization Algorithm

In this section we present a summary of the experiments carried out to select the minimization algorithm. Since our primary interest was to isolate the effects of the adaptive horizontal controller, the action of the vertical controller was limited to injecting enough energy into the system to maintain hopping but without any attempt to control the hopping height.

Two classes of algorithms were tested for the on-line minimization:

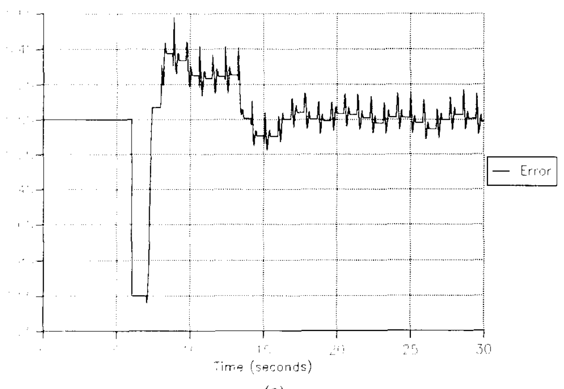
1) random search techniques
2) deterministic minimization techniques.

The main comparison criteria were the number of function evaluations required to reach the minimum with a given accuracy and the robustness of the algorithm, that is, its ability to handle large variations in the nominal parameters and to reject spurious disturbances.

Random Search Algorithms: For a function of one variable, the number of function evaluations needed to reach the minimum is known to be greater for random search techniques than for the classical, deterministic techniques such as gradient searches [11]. However, random search algorighms are especially suited for the case where the structure of the objective function is unknown and where the relevant penalty involved in the search is associated with the number of function evaluations required to reach the minimum [11]. Another appealing feature of these algorithms is that they are simple and easily implemented.

From this class of algorithms we selected the Adaptive Step Size Random Search (ASSRS), proposed by Schumer and Steiglitz [11], as an approximation to a hypothetical algorithm that always uses the optimal step size for the random search. In the ASSRS algorithm, the search for the minimizing parameter is conducted by a process of trial and error. In the general case of an n-dimensional minimization, a random direction, distributed uniformly over an n-dimensional hypersphere is selected. Starting from the last approximation to the minimum, a two-step search is conducted in the selected direction. Let s be the step size selected during the last search as an approximation to the optimal step size. The points located at a distance s and $s * (1 + a)$ in the selected direction, where a is a uniform random variable in $[-0.5, 0.5]$, are tested as candidates for the new approximation to the minimum. The point that yields the best improvement over the current optimum is adopted as the new guess and its distance from the previous guess becomes the new step size. If neither of the points yields an improvement a new direction is selected and the search resumes. If m consecutive searches (where m is a parameter of the algorithm) have been unsuccessful, the step size is halved and the algorithm starts earching again using the new step size. To prevent the algorithm from locking into a local minimum due to the step size becoming too small, a larger step is tested after a large number of iterations has passed. The complete algorithm particularized for the one-dimensional case is listed in [10].

Fig. 6(a) and (b) shows the results of the simulation of the ASSRS algorithm for a step of 1 m/s in the reference velocity applied to the system at $t = 6$. From $t = 6$ to $t = 13$ only the basic controller is acting. At $t = 13$ the system is close enough to a steady state and the adaptive controller begins to act. The plots show that the error



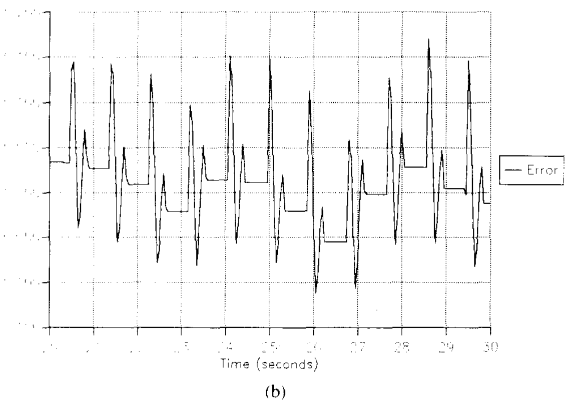


Fig. 7. (a) Velocity error for the Newton–Raphson algorithm. (b) Velocity error for the Newton–Raphson algorithm.

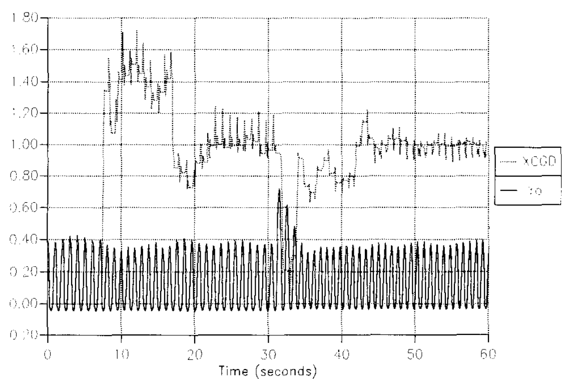


Fig. 8. Response of the ASSRS algorithm to a 30-percent decrease in mass.

decays very quickly and remains in the order of 0.01 m/s, compared to an error on the order of 0.2 m/s when only the basic controller is utilized.

Deterministic Minimization Algorithms:

a) Gradient search algorithms: Fig. 5 shows that the square of the velocity error as a function of the correction term K_{corr} is unimodal and approximately quadratic in the neighborhood of the minimum. Thus we can expect gradient minimization algorithms such as the Newton gradient method to converge rapidly, at least in the local region of the minimum. The difficulty with these methods is that the gradient of the function (and also the Hessian for the Newton method) must be estimated numerically since the functional form is unknown. This numerical estimation causes problems in the immediate neighborhood of the minimum where the increments in the independent variable generated by the algorithm become increasingly smaller and hence being greatly influenced by the error in the gradient estimation. This difficulty can be partially overcome using an alternative formulation for the problem [10].

The response of the controller employing the Newton–Raphson algorithm for the numerical minimization to a step of 1 m/s in the reference velocity is shown in Fig. 7(a) and (b). The plots show that once the adaptive controller is applied (at $t = 13$ s) the velocity error decays quickly. In the amplification of the last part of the trajectory, shown in Fig. 7(b), we see that the remaining error is of the order of 0.01 m/s compared with an error of 0.2 m/s when using only the basic controller. However, note that the trajectory is irregular in the sense that at some instants the error increases and then decreases rapidly. This irregularity is caused by the errors in the numerical estimation of the gradient when the algorithm gets close to the minimum [10].

b) Other sequential search algorithms: As an alternative to the gradient type techniques, we explored a class of algorithms that start from an interval that is known to contain the minimizing parameters and generate a sequence of subintervals that also contains the parameter and converges to zero length. We found that, although these algorithms converged quickly, they were extremely sensitive to the initial guess for the interval that brackets the minimum [10]. We can conceivably provide a good initial guess for the required interval for a nominal set of parameters of the robot. Hence, the procedure will work well enough for small variations of the parameters around their nominal value, but it will severely limit the adaptability of the robot to larger variations. Therefore, this class of algorithms is not well suited for our application.

Based upon the series of experiments presented in this section, we selected the ASSRS algorithm for the on-line minimization. As expected, the Newton–Raphson algorithm converges faster, but it had difficulties associated with the numerical estimation of the gradient. Further, due to this numerical estimate, we can expect the algorithm to be only moderately robust. A spurious disturbance can cause a false estimate for the gradient and drive the algorithm far away from the true minimum. On the other hand, the ASSRS algorithm converges almost as fast as the Newton–Raphson without the problems associated with the numerical estimate of the gradient. Furthermore, since the algorithm tests two different step sizes in each direction, the second one being random, any spurious disturbance will not have an effect as pronounced as in the Newton–Raphson case.

IV. Examples of the Performance of the Adaptive Controller

In this section we present some examples of the performance of the adaptive controller, using the ASSRS algorithm, for different situations. Since some of the cases involve rather large departures of the robot parameters from their nominal values we needed a vertical control algorithm which could control accurately the hopping height. Hence we employed an algorithm different from the one proposed by Raibert [8], although based upon the same idea. This new algorithm uses a different estimate for the energy to be injected into the system that takes into account the finite time interval needed to expand the leg. This estimate is combined with a linear feedback of the hopping height error in previous hops. The algorithm is described in the Appendix.

Fig. 8 shows the response to a decrease in the mass for our standard input (a step of 1 m/s in the reference velocity, applied at $t = 6$ s). The initial mass is 12 kg, 20 percent higher than the nominal value and represents a robot of 8 kg loaded with a payload of 4 kg. During the first stance period after $t = 30$ s the mass is decreased to 8 kg, modeling the drop of the load. Fig. 9 also shows the response for a change in mass. At $t = 0$ the mass is set to 8 kg, 20 percent lower than the nominal value. During the first stance period after $t = 30$ s the mass is increased to 12 kg, representing the robot picking up some payload.

Fig. 10 shows the response to a change in the characteristics of the ground model. At $t = 0$ the ground model constants are set

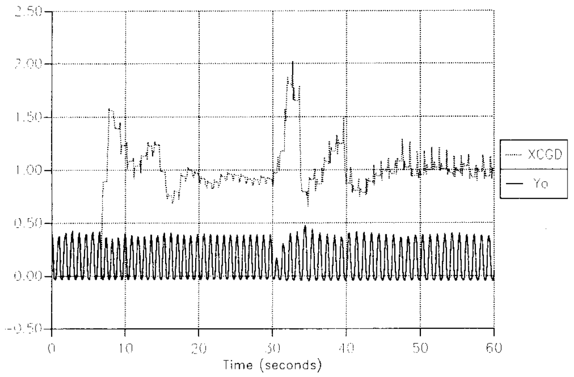


Fig. 9. Response of the ASSRS algorithm to a 30-percent increase in mass.

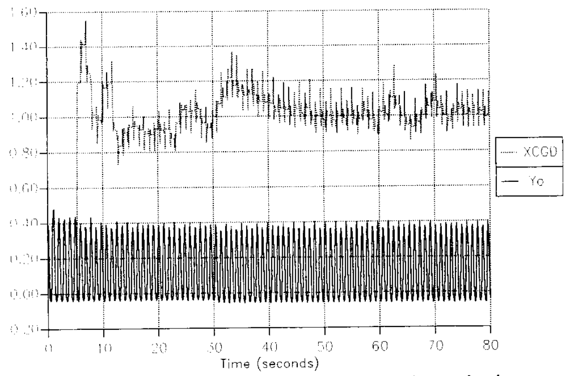


Fig. 10. Response of the ASSRS algorithm to a change in the ground model.

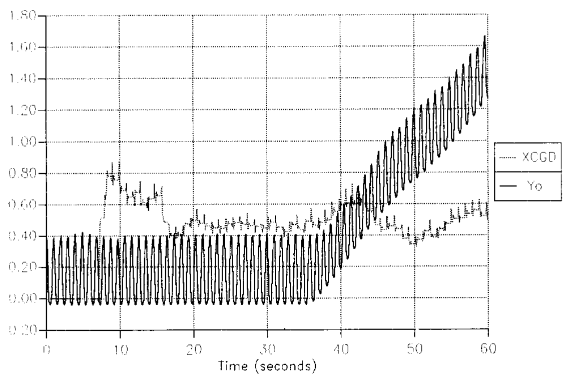


Fig. 11. Response of the ASSRS algorithm to a change in slope.

to their nominal values. At $t = 30$ s their values are changed to $K_G = 0.7 * 10^4$ N/m and $B_G = 100$ N·s/m modeling a softer, lossier terrain.

Our last experiment, Fig. 11, shows the robot going up a slope. The robot starts moving on level ground and at $x = 15$ m, measured from the starting position, the ground is changed to a slope of 6°.

This series of experiments confirms the fact that our adaptive controller is capable of controlling the robot for a wide range of variations in the values of the parameters, variations that model some of the tasks relevant to a dynamically stabilized walking machine.

V. Conclusion

This communication presents an application of adaptive control based upon the numerical minimization of a performance criteria. Specifically, we seek to replace an existing controller for a dynamically stabilized legged machine with an improved adaptive controller. This adaptive controller eliminates the problem of a bias in the forward velocity experienced with the nonadaptive controller and, at the same time, provides the flexibility required to allow a dynamically stabilized legged machine to perform satisfactorily under the widely varying conditions that exist in the real world. We believe that adaptive controllers based upon on-line numerical minimization may prove of great importance in robotics problems where the complete dynamics of the plant is either unknown or too complex to work with. In particular, based on our experience with the one-legged robot, we feel that random search algorithms may prove to be a valuable tool in such cases, outperforming other minimization algorithms.

Appendix

The Vertical Controller

In this Appendix we give a brief description of the fundamentals of the vertical controller used for the examples of Section IV. We will use the model developed in Section II-C to compute the amount of energy to be injected into the system in order to achieve the desired hopping height.

Assume that the leg is extended, starting at touchdown, using the linear law

$$X(t) = at. \tag{A1}$$

Then the solution for the motion is given by

$$y(t) = -(v_0 + a)(1/w_m)\sin w_m t + at + l_0 \tag{A2}$$

where v_0 is the velocity immediately after touchdown and $w_m = \sqrt{k/m}$. The kinetic and potential energies of the system immediately before liftoff are given by

$$T_{\mathrm{LO}_-} = 0.5m(v_0 + 2a)^2 \cong 0.5mv_0^2 + 2mv_0 a \tag{A3}$$

$$U = mgX(t_{\mathrm{LO}}) = mgaT_{\mathrm{ST}} \tag{A4}$$

where t_{LO} is the liftoff instant and T_{ST} is the total duration of the stance period.

Generalizing these results to the actual robot and applying conservation of linear momentum we have that the total energy immediately after liftoff is

$$E_{\mathrm{LO}} = 0.5M_2v_0^2 + 2M_2v_0a + (M_1 + M_2)gaT_{\mathrm{ST}}. \tag{A5}$$

At the highest point of the trajectory, all the energy is in the form of potential energy so the maximum height achieved in the next hop, $\hat{H}_{k+1}$, is given by

$$\hat{H}_{k+1} = E_{\mathrm{LO}}/(M_1 + M_2)g. \tag{A6}$$

By similar considerations, the velocity immediately after touchdown is related to the maximum height achieved in the previous hop, H_k, by

$$v_0^2 = 2gH_kM_2/(M_1 + M_2). \tag{A7}$$

From (A5)–(A7) we can obtain a relationship between the variable a (our control input), the height achieved in the last hop H_k, and the desired height for the next hop $\hat{H}_{k+1}$

$$a = (\hat{H}_{k+1} - R_m^2H_k)/(T_{\mathrm{ST}} + \sqrt{8R_m^3H_k/g}),$$

$$\text{where } R_m = M_2/(M_2 + M_1). \tag{A8}$$

Combining this control law with a linear feedback of the hopping height error we finally get

$$X(t) = a'(t - t_{\mathrm{TD}}) \tag{A9}$$

where

$$a' = a + K_{\mathrm{vert}}(\hat{H}_k - H_k) + K_{\mathrm{int}}S$$
$$S = \Sigma_{j=1}^{k-1}(\hat{H}_j - H_j)$$

H_k	actual maximum height at the k hop
$\hat{H}_k$	desired height at the k hop
K_{vert}, K_{int}	gain constants (0.1 in our simulation)
t_{TD}	touchdown instant.

References

[1] C. A. Klein, K. W. Olson, and D. R. Pugh, "Use of force and attitude sensors for locomotion of a legged vehicle over irregular terrain," *Int. J. Robotics Res.*, vol. 2, no. 2, pp. 3-17, 1983.

[2] R. B. McGhee, F. Ozuguner, and S. J. Tsai, "Rough terrain locomotion by a hexapod robot using a binocular ranging system," in *Robotics Research,* M. Brady and R. P. Paul, Eds. Cambridge, MA: MIT Press, 1984, pp. 227-251.

[3] M. H. Raibert and I. E. Sutherland, "Machines that walk," *Scientific Amer.*, vol. 248, no. 1, pp. 44-53, 1983.

[4] M. H. Raibert, *Legged Robots That Balance.* Cambridge, MA: MIT Press, 1986.

[5] M. H. Raibert, H. B. Brown, and S. S. Murthy, "3-D balance using 2-D algorithms?" in *Robotics Research,* M. Brady and R. P. Paul, Eds. Cambridge, MA: MIT Press, 1983, pp. 279-301.

[6] M. H. Raibert and F. C. Wimberly, "Tabular control of balance in a dynamic legged system," *IEEE Trans. Syst,. Man. Cybern.*, vol. SMC-14, no. 2, pp. 334-339, 1984.

[7] M. H. Raibert, "Hopping in legged systems. Modeling and simulation for the two-dimensional one-legged case," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-14, no. 3, pp. 451-463, 1984.

[8] M. H. Raibert, H. B. Brown, and M. Chepponis, "Experiments in balance with a 3D one-legged hopping machine," *Int. J. Robotics Res.*, vol. 3, no. 2, pp. 75-92, 1984.

[9] M. H. Raibert, M. Chepponis, and H. B. Brown, "Running on four legs as though they were one," *IEEE J. Robotics Automat.*, vol. RA-2, no. 2, pp. 70-82, 1986.

[10] M. Sznaier, "An adaptive controller for a dynamically stabilized one-legged walking machine," Master thesis, University of Washington, Seattle, 1986.

[11] M. A. Schumer and K. Steiglitz, "Adaptive step size random search," *IEEE Trans. Automat. Contr.*, vol. AC-13, no. 3, pp. 270-276, 1968.