Robust Controller Design for a Constrained Flexible Structure via Mixed l_∞/H_∞ Optimization

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Abstract

A successful controller design paradigm must take into account both model uncertainty and performance specifications. Model uncertainty can be addressed using the $\mathcal{H}_\infty$ robust control framework. However, this framework cannot accommodate the realistic case where in addition to robustness considerations, the system is subject to time domain specifications. We recently proposed a design procedure to explicitly incorporate time–domain specifications into the $\mathcal{H}_\infty$ framework [1]. In this paper we apply this design procedure to the simple flexible structure used as a benchmark in the 1990–1992 ACC, with the goal of minimizing the peak control effort due to disturbances while satisfying settling time and robustness specifications. The results show that there exist a severe trade–off between peak control action and robustness to unstructured model uncertainty.

1. Introduction

A substantial number of control problems, spanning applications as diverse as thermo–electric generating plants, robotic systems and large space structures, can be summarized as the problem of designing a controller capable of achieving acceptable performance under system uncertainty and design constraints. This statement looks deceptively simple, but even in the case where the system under consideration is linear, the problem is far from solved. During the last decade a large research effort has been devoted to the problem of designing "robust" controllers, capable of achieving desirable properties under various classes of plant uncertainties while, at the same time, satisfying frequency–domain constraints. As a result, a powerful framework has been developed, addressing the issues of robust stability and robust performance in the presence of norm–bound uncertainties by minimizing a weighted $\mathcal{H}_\infty$ norm [2,3]. The $\mathcal{H}_\infty$ formalism has gained wide acceptance, since it embodies many desirable design objectives. Further, in conjunction with μ–analysis [4], it has been successfully applied to a number of hard practical control problems (see for instance [5]). However, in spite of this success, it is clear that plain $\mathcal{H}_\infty$ control can only address a subset of the common performance requirements since, being a frequency domain method, it can not address time domain specifications. Recently some progress has been made in this direction [6–9], but most of the proposed methods rely on a number of approximations, which may preclude finding a solution if the design specifications are tight.

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A different approach to robust control has been pursued in [10–13], where robustness and disturbance rejection are approached using the l_1 optimal control theory introduced by Vidyasagar [10] and developed by Pearson and coworkers [11–13]. These methods are attractive since they allow for an explicit solution to the robust performance problem. However, they cannot accommodate some common classes of frequency domain specifications (such as $\mathcal{H}_2$ or $\mathcal{H}_\infty$ bounds).

In recent papers [1, 14] we addressed the problem of finding an internally stabilizing compensator that minimizes the maximum amplitude of the output to a fixed given input, subject to constraints upon the $\mathcal{H}_\infty$ norm of a relevant transfer function. In this paper we apply this framework to the problem of designing a controller for the 1990-1992 ACC Robust Control Benchmark Problem capable of achieving minimum peak control effort while, at the same time, maintaining an adequate robustness level against model uncertainty. This simple flexible structure example highlights both the strength of the method, namely the ability to explicitly identify the trade–off between time and frequency domain specifications, and its main disadvantage, the fact that it results in high order controllers that may necessitate some form of model reduction. Thus, at this stage the main contribution of the framework is to serve as a benchmark, indicating the limits of performance imposed by the plant, rather than providing a practical design tool.

The paper is organized as follows: In section II we introduce the mixed l_∞/H_∞ optimization problem and we briefly review the solution method presented in [1]. The main result of this section shows that the mixed optimization problem can be *exactly* solved by using a two step procedure that involves solving first a finite dimensional convex, albeit in general non–differentiable, optimization problem, and then solving an *unconstrained* Nehari approximation. In section III we present a simple design example and we compare our controller to the unconstrained optimal $\mathcal{H}_\infty$ controller. In section IV we apply the framework to design a controller for a simple flexible structure, the ACC Robust Control Benchmark Problem. Finally, in section V, we summarize our results and we indicate directions for future research.

2. Problem Formulation.

2.1 Notation

By $\mathcal{L}_\infty$ we denote the Lebesgue space of complex valued transfer matrices which are essentially bounded on the unit circle with norm $\|T(z)\|_{\mathcal{H}_\infty} \stackrel{\Delta}{=} \sigma_{\max}(T(e^{j\omega}))$.$\mathcal{H}_\infty$ ($\mathcal{H}_\infty{}^-$) denotes the set of stable (antistable) complex matrices $g(z) \in \mathcal{L}_\infty$, i.e analytic in $z \geq 1$ ($z \leq 1$). $\mathcal{RH}_\infty$ ($\mathcal{RH}_\infty^-$) denotes the subset of $\mathcal{H}_\infty$ ($\mathcal{H}_\infty{}^-$) formed by real rational transfer matrices. l_∞ denotes

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the space of bounded real sequences $\{e_k\}$ equipped with the norm $\|e\|_{l_\infty} \triangleq \sup_k |e_k|$. To avoid confusion, we will denote the $\mathcal{H}_\infty$ norm of a transfer function as $\|.\|_{\mathcal{H}_\infty}$ and the l_∞ norm of a sequence as $\|.\|_{l_\infty}$. Throughout the paper we will use packed notation to represent state-space realizations, i.e.

$$G(z) = C(zI - A)^{-1}B + D \triangleq \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$$

For a transfer matrix $G(z)$, $G^{\sim} \triangleq G'(\frac{1}{z})$ where $'$ indicates transpose conjugate. Finally, $\underline{x}$ indicates that x is a vector quantity.

2.2 Statement of the Problem

Consider the system represented by the block diagram 1, where the scalar signals v, w and u represent an exogenous disturbance, a *known, fixed* signal, and the control action respectively; ζ and ψ represent the outputs subject to frequency and time domain performance specifications respectively; and y represents the measurements available to the controller. Note that v and ζ include fictitious signals used to assess stability in the presence of model uncertainty. Then, the *mixed $l_\infty/\mathcal{H}_\infty$ problem* can be stated as follows:

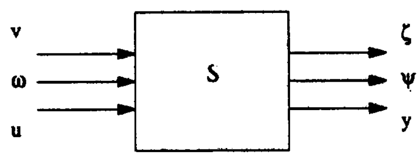

Figure 1: Block Diagram of the Generalized Plant.

Given the nominal system (S), with frequency-domain performance specifications of the form:

$$\|T_{\zeta v}\|_{\mathcal{H}_\infty} \leq \gamma \qquad (P)$$

find an internally stabilizing controller

$$u(z) = K(z)y(z) \qquad (C)$$

such that the maximum amplitude of the regulated output ψ due to ω is minimized subject to the performance specifications (P)

2.3 Problem Solution

In this section we briefly review the framework presented in [1]. The main result of this section shows that the mixed $l_\infty/\mathcal{H}_\infty$ optimization problem can be decoupled into a constrained convex finite dimensional optimization and an unconstrained Nehari extension problem. The key to this result is to i) use the Youla [15] parametrization of all stabilizing controllers to transform the problem into a constrained convex optimization problem ii) expand the free parameter q into a power series and iii) observe that only the first N (where N depends on the problem but can be determined before hand) terms of this expansion appear in the optimization of the time response. These results are summarized in the following theorems:

• **Theorem 1:** **The set of all closed-loop transfer matrices achievable by an internally stabilizing compensator can be parametrized in terms of a free parameter $Q \in \mathcal{RH}_\infty$ as:**

$$\begin{aligned} T_{\zeta v} &= T_{11} + T_{12}QT_{21} \\ T_{\psi w} &= T_{11}^{\psi} + T_{12}^{\psi}QT_{21}^{\psi} \end{aligned} \qquad (1)$$

where $T_i, T_i^{\psi} \in \mathcal{RH}_\infty$. Moreover, it is possible to select the parametrization in such a way that $T_{12}(z)$ and $T_{21}(z)$ are inner and co-inner respectively (i.e. $T_{12}^{\sim}T_{12} = I$, $T_{21}T_{21}^{\sim} = I$).

Proof: These results are well known and have been proved in several different ways. See for instance [3,16] for a proof using an observer-based argument.

Remark 1: For the SISO case, equation (1) reduces to:

$$\begin{aligned} T_{\zeta v}(z) &= t_1(z) + t_2(z)q(z) \\ T_{\psi w}(z) &= t_1^{\psi}(z) + t_2^{\psi}(z)q(z) \end{aligned} \qquad (2)$$

where t_i, t_i^{ψ}, q are stable transfer functions and where t_2 is inner. Since $\|.\|_{\mathcal{H}_\infty}$ is invariant under multiplication by an inner function we have:

$$\|T_{\zeta v}\|_{\mathcal{H}_\infty} = \|t_1 + t_2 q\|_{\mathcal{H}_\infty} = \|t_1 t_2^{\sim} + q\|_{\mathcal{H}_\infty} = \|R + q\|_{\mathcal{H}_\infty} \qquad (3)$$

where $R(z) \triangleq t_1(z)t_2^{\sim}(z)$ has all its poles outside the unit disk.

By using this parametrization the mixed optimization problem can be now precisely stated as solving:

$$\mu^c = \inf_{q \in \mathcal{RH}_\infty} \|\psi_k\|_{l_\infty} \qquad (OPT)$$

subject to:

$$\|t_1(z) + t_2(z)q(z)\|_{\mathcal{H}_\infty} \leq \gamma$$

where

$$\psi_k = Z^{-1}\left\{\Psi(z) = (t_1^{\psi}(z) + t_2^{\psi}(z)q(z))\omega(z)\right\}$$

Problem (OPT) is a convex optimization problem in $\mathcal{RH}_\infty$. However, since this space is not compact, a minimizing solution may not exist. Moreover, even when a solution does exist, it may yield a system with extremely large settling time. These difficulties can be avoided by constraining the poles of the closed-loop system to lie in a disk with radius $\delta < 1$. Thus, rather than solving the original problem (OPT) it is convenient to solve the following modified problem (OPT_δ):

$$\mu_c^{\delta} = \min_{q \in \mathcal{RH}_\infty^{\delta}} \|\psi\|_{l_\infty} \qquad (OPT_\delta)$$

subject to:

$$\begin{gathered} \|t_1(z) + t_2(z)q(z)\|_{\mathcal{H}_\infty^{\delta}} \leq \gamma \\ t_i,\ t_i^{\psi},\ q \text{ analytical in } |z| \geq \delta \end{gathered}$$

where $\delta < 1$, $\mathcal{RH}_\infty^{\delta} = \{q(z) \in \mathcal{RH}_\infty : q(z) \text{ analytical in } |z| \geq \delta\}$, and where $\|q\|_{\mathcal{H}_\infty^{\delta}} \triangleq \sup_{|z|=\delta} |q(z)|$.

Note that from the maximum modulus theorem, $\|T_{\zeta v}\|_{\mathcal{H}_\infty^{\delta}} \geq \|T_{\zeta v}\|_{\mathcal{H}_\infty}$. Thus, a solution to (OPT_δ) is guaranteed to satisfy the original constraints (C). It follows that μ_c^{δ} is an upper bound of μ_c. In the sequel, we show that (OPT_δ) can be solved by solving first a finite dimensional optimization problem and then solving an unconstrained Nehari optimization problem. We begin by showing that the minimization of $\|\psi_k\|_{l_\infty}$ subject to the constraints (P) requires considering only a finite number N of elements of the sequence ψ_k.

• **Theorem 2:** Assume that the mixed optimization problem is feasible and let q^*, ψ^* denote the solution. Then, there exist a finite number N such that:

$$\begin{aligned} \mu_c = \|\psi^*\|_{l_\infty} &= \sup_{0\le k\le N-1} |\psi_k^*| \triangleq \|\underline{t}_1 + \tau \underline{q}^*\|_\infty \\ |\psi_k^*| < \mu_c \qquad & k \ge N \end{aligned} \tag{4}$$

where:

$$\begin{aligned} \underline{t}_1 &\triangleq (\, t_{1o} \;\; \dots \;\; t_{1N-1}\,)' \\ \tau &= \begin{pmatrix} t_{2o}^{\tau} & 0 & \dots & 0 \\ t_{21}^{\tau} & t_{20}^{\tau} & \dots & 0 \\ \vdots & & \ddots & \\ t_{2N-1}^{\tau} & \dots & & t_{2o}^{\tau} \end{pmatrix} \\ \underline{q}^* &\triangleq (\, q_o \;\; \dots \;\; q_{N-1}\,)' \end{aligned} \tag{5}$$

and where t_{ik} denotes the k^{th} element of the impulse response of $t_i^{\psi}(z)\omega(z)$ (i.e. $t_i^{\psi}(z)\omega(z) = \sum_0^\infty t_{ik} z^{-k}$)

Proof: See [1].

• **Theorem 3:** Let $q_F = \sum_{i=0}^{N-1} q_i z^{-i} \in \mathcal{RH}_\infty$ be given. Then, there exist $q_R \in \mathcal{RH}_\infty$ such that $\|R + q_F + z^{-N} q_R\|_{\mathcal{H}_\infty} \le \gamma$, where *iff* $\|Q\|_2 \le \gamma$ where:

$$\begin{aligned} Q &= W^{\frac12}\begin{pmatrix} I & 0 \\ 0 & \mathcal{H}' \end{pmatrix} L_c^{\frac12} \\ Lc &= \begin{pmatrix} L_{11}^C & L_{12}^C \\ L_{12}^{C\,\prime} & L_{22}^C \end{pmatrix} \\ L_{11}^C &= L_o^C \\ L_{12}^C &= -\left((A_R')^{N-1} c_R' \quad (A_R')^{N-2} c_R' \dots \quad c_R' \right) \\ L_{22}^C &= I_N \\ W'^{\frac12} W^{\frac12} &= \begin{pmatrix} L_o^0 & \mathcal{A} \\ \mathcal{A}' & I \end{pmatrix} \\ \mathcal{A} &= \left(A_R^{-N} b_R \quad A_R^{-(N-1)} b_R \dots A_R^{-1} b_R \right) \\ \mathcal{H} &= \begin{pmatrix} h_N & h_{N-1} & \dots & \dots & h_1 \\ & h_N & h_{N-1} & \dots & h_2 \\ & & \ddots & & \\ & & & h_N & h_{N-1} \\ & & & & h_N \end{pmatrix} \\ h_i &= q_{N-i} + b_R' (A_R')^{N-1-i} c_R' \qquad 1 \le i \le N-1 \\ h_N &= q_0 + d_R \\ R &= \left(\begin{array}{c|c} A_R & b_R \\ \hline c_R & d_R \end{array} \right) \end{aligned} \tag{6}$$

and where L_o^0 and L_o^C are the solutions to the following Lyapunov equations:

$$\begin{aligned} A_R L_o^0 A_R' - L_o^0 &= b_R b_R' \\ A_R' L_o^C A_R - L_o^C &= (A_R')^N c_R' c_R (A_R)^N \end{aligned} \tag{7}$$

Proof: Let $G \triangleq R + q_F$. The proof follows by noting that, given q_F, there exist $q_R \in \mathcal{RH}_\infty$ such that $\|T_{\zeta w}\|_{\mathcal{H}_\infty} \le \gamma$ *iff* the corresponding *unconstrained* 1 block Nehari approximation problem has a solution, i.e. if:

$$\begin{aligned} \min_{q_R \in \mathcal{RH}_\infty} \|G + z^{-N} q_R\|_{\mathcal{H}_\infty} &= \min_{q_R \in \mathcal{RH}_\infty} \|z^N G + q_R\|_{\mathcal{H}_\infty} \\ &= \min_{q_R \in RH_\infty^-} \|z^{-N} G^{\sim} + q_R\|_{\mathcal{H}_\infty} \\ &= \Gamma_H(z^{-N} G^{\sim}) \le \gamma \end{aligned} \tag{8}$$

where Γ_H indicates the maximum Hankel singular value and where we used the facts that z^N is an inner function and that the best stable approximation to a given function coincides with the best antistable approximation to its conjugate. In order to compute Γ_H we need to compute the observability L_o and controllability L_c grammians of the stable part $\mathcal{G}$ of $z^{-N}G^{\sim}$. In [1] we showed, through some lengthy computations, that these grammians can be computed explicitly. Furthermore, L_c is independent of q_F and L_o is given by:

$$\begin{aligned} L_o &= \begin{pmatrix} L_o^0 & \mathcal{A}\mathcal{H}' \\ \mathcal{H}\mathcal{A}' & \mathcal{H}\mathcal{H}' \end{pmatrix} \\ &= \begin{pmatrix} I & 0 \\ 0 & \mathcal{H} \end{pmatrix} \begin{pmatrix} L_o^0 & \mathcal{A} \\ \mathcal{A}' & I \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & \mathcal{H}' \end{pmatrix} \end{aligned}$$

Hence:

$$\begin{aligned} L_c^{\frac12} L_o L_c^{\frac12} &= Q'Q \\ Q &\triangleq W^{\frac12} \begin{pmatrix} I & 0 \\ 0 & \mathcal{H}' \end{pmatrix} L_c^{\frac12} \end{aligned} \tag{9}$$

From Nehari Theorem it follows that:

$$\begin{aligned} \|T_{\zeta w}\|_{\mathcal{H}_\infty} \le \gamma &\iff \rho^{\frac12}\left(L_c^{\frac12} L_o L_c^{\frac12} \right) \le \gamma \\ &\iff \|Q\|_2 \le \gamma \end{aligned} \tag{10}$$

where ρ indicates the spectral radius. Since Q is a linear function of the coefficients of q_F it follows that the constraint (10) is convex in the variables q_i ⋄.

Remark 2: The results of Theorem 3 can be applied to the constraint $\|T_{\zeta v}\|_{\mathcal{H}_\infty^\delta} \le \gamma$ by using the change of variable $z = \delta \hat{z}$ to map the δ–disk to the unit disk.

The following result is now obvious:

• **Theorem 4:** $q^o = q_F^o + z^{-N} q_R^o$ solves the mixed $l_\infty/\mathcal{H}_\infty$ control problem iff $\underline{q}^o = (\, q_F^o \dots q_{N-1}\,)'$ solves the following finite dimensional convex optimization problem:

$$\underline{q}_F^o = \operatorname*{argmin}_{\substack{\underline{q} \in R^N \\ \|Q\|_2 \le \gamma}} \|\underline{t}_1 + \tau \underline{q}\|_\infty \tag{11}$$

and q_R^o solves the unconstrained Nehari approximation problem

$$q_R^o = \operatorname*{argmin}_{q_R \in \mathcal{RH}_\infty^\delta} \|R + q_R\|_{\mathcal{H}_\infty^\delta} \tag{12}$$

where R is defined in Theorem 1.

Remark 3: From the results of Theorem 4, it follows that the mixed optimization problem can be solved by using the following algorithm: i) Use the transformation $z = \delta\hat{z}$ to map the δ–disk to the unit disk, ii) solve the convex finite dimensional optimization (11); iii) solve the unconstrained Nehari approximation problem (12), iv) use the transformation $\hat{z} = \delta^{-1} z$ to obtain the controller and the closed–loop system.

3. A Simple Example

Consider the problem of minimizing the step response error for the non-minimum phase plant shown in figure 2. Furthermore, assume that the system is subject to unstructured multiplicative uncertainty as shown in figure 2. Table 1 shows $\|\psi\|_{l_\infty}$ and $\|T_{\zeta v}\|_{\mathcal{H}_\infty}$ for different designs, with the corresponding step and frequency responses shown in figure 3. By using the results of [13, Theorem 4], it can be easily shown that the infimum of the error is $\|\psi\|_{l_\infty} = \frac{8}{3}$, achieved with the controller $C(z) = \frac{z-1}{z}$. The same controller yields $\|T_{\zeta v}\|_{\mathcal{H}_\infty} = 5$ thus guaranteeing robust stability against unstructured perturbations $\|\Delta\|_{\mathcal{H}_\infty} \leq 0.2$. Note that this controller is *not internally stabilizing* due to the pole–zero cancellation at $z = 1$. The optimal $\mathcal{H}_\infty$ controller yields $\|T_{\zeta v}\|_{\mathcal{H}_\infty} = 3$ and $\|\psi\|_{l_\infty} = 4$. Mixed $l_\infty/\mathcal{H}_\infty$ optimization with $\|T_{\zeta w}\|_{\mathcal{H}_\infty} \leq 3.3$ yields $\|\psi\|_{l_\infty} = 3.31$. However, this procedure results in a controller with 153 states. Finally, the last entry in Table 1 corresponds to a reduced-order controller with 5 states. In spite of the substantial order reduction, this controller yields virtually the same performance as the mixed $l_\infty/\mathcal{H}_\infty$ controller.

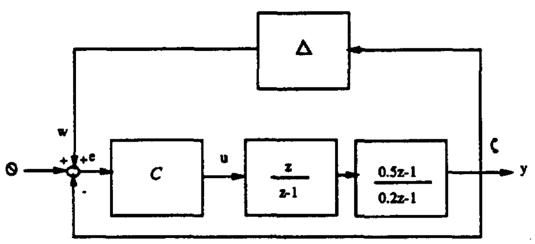

Figure 2. Block Diagram with Multiplicative Uncertainty Δ "Pulled–Ou

	$\|T_{\zeta v}\|_{\mathcal{H}_\infty}$	$\|\psi\|_{l_\infty}$
l_∞	5	$\frac{8}{3}$
$\mathcal{H}_\infty$	3	4
$l_\infty/\mathcal{H}_\infty$	3.3	3.311
$l_\infty/\mathcal{H}_{\infty\,\mathrm{red}}$	3.3	3.312

Table 1. $\|T_{\zeta v}\|_{\mathcal{H}_\infty}$ vs $\|\psi\|_{l_\infty}$ for the Simple Example

4. Controller Design for the ACC Benchmark Problem

The issues involved in controlling systems subject to model uncertainty and constraints can be illustrated by the simple system shown in figure 4, consisting of two unity masses coupled by a spring with constant $0.5 \leq k \leq 2$ but otherwise unknown. A control force acts on body 1 and the position of body 2 is measured, resulting in a non–colocated sensor actuator problem that embodies many of the pathologies and challenges present in realistic problems, such as control of complex aircraft and large space structures [17]. This system has been used as a benchmark during the last few years at the American Control Conference [18–19] to highlight the issues and trade–offs involved in robust control design.

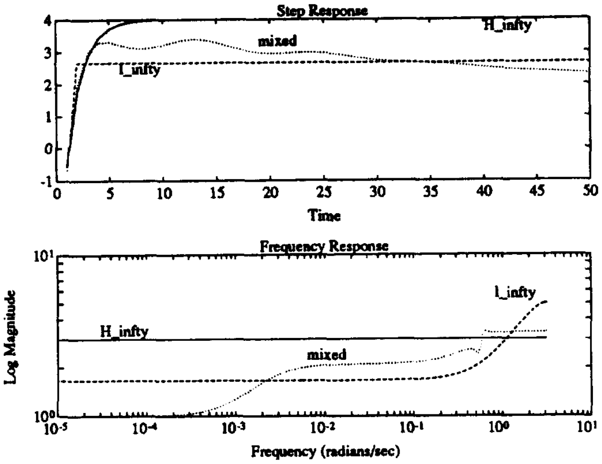

Figure. 3. Step and Frequency Responses for Different Designs.

Consider the problem of designing a stabilizing controller subject to the following performance specifications: i) the closed-loop system must be stable for all possible values of the uncertain parameter k. ii) the peak of the control action following a unit impulse disturbance ω acting on m_2 should be minimized; and iii) for the same disturbance the displacement y of m_2 has a settling time of about 15 seconds.

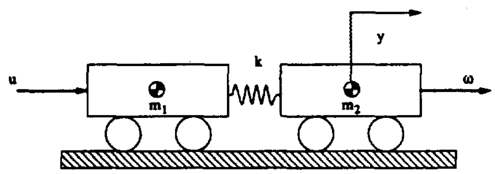

Figure 4: The ACC Robust Control Benchmark Problem.

In order to fit the problem into the $\mathcal{H}_\infty$ framework, the uncertain spring constant k is modeled as $k = k_o + \Delta$ (with $k_o = 1.25$ and $\|\Delta\| \leq 0.75$) and, following a standard procedure [16], Δ is "pulled out" of the system, as shown in figure 5. The problem can be stated now as the problem of minimizing the peak control effort u_{pk} over the set of all internally stabilizing controllers, subject to the settling time and $\|T_{\zeta v}\|_{\mathcal{H}_\infty} \leq \frac{4}{3}$ constraints.

The system, with the uncertainty "pulled out", can be represented by the following state space realization:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_o & k_o & 0 & 0 \\ k_o & -k_o & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

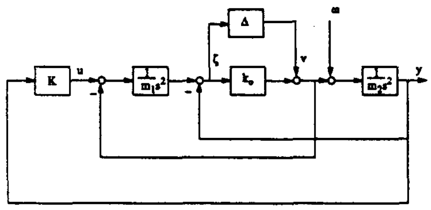

Figure 5 Block Diagram with the Uncertainty "Pulled Out" of the System.

In order to fit the problem into our framework, the system was discretized using sample and hold elements at the inputs and outputs, with a sampling time of 0.1 seconds.

The need to explicitly take into account the peak of the control action is illustrated in figure 6, showing the frequency and impulse responses obtained with a controller obtained using the standard state–space $\mathcal{H}_\infty$ design procedure [3]. This controller achieves $\|T_{\zeta v}\|_\infty = 1.1$ (hence satisfying the robustness constraint), however it results in a *clearly unrealistically large* peak control action.

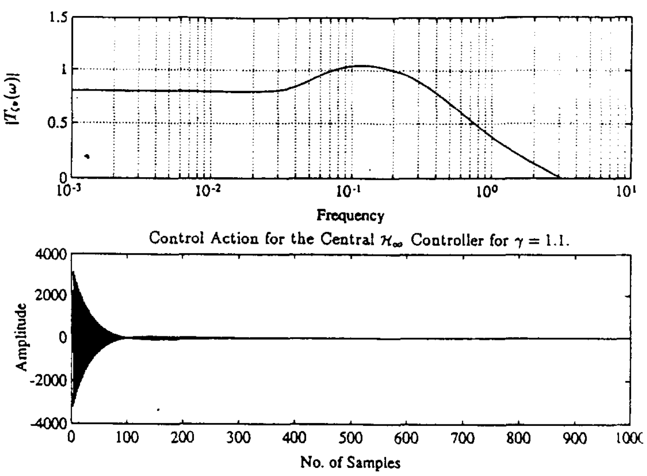

Figure 6. Frequency Response and Control Action for an Unconstrained Controller.

Figure 7 shows the peak control action versus $\|T_{\zeta v}\|$, subject to the settling time constraint. From the figure, it follows that there exist a severe trade–off between peak control action and robustness to unstructured dynamic uncertainty. In particular, achieving $\|T_{\zeta v}\|_{\mathcal{H}_\infty} \leq \frac{4}{3}$, requires a peak control action of approximately 1. Hence, for the discretized version of the BMP, the specifications are (barely) achievable, although they may require a very large order controller. It should be noted that the settling time constraint is binding. By slightly relaxing this constraint, the specifications are achievable with the the following second order controller:

$$Ac = \begin{pmatrix} 1.7404 & -0.7769 \\ 0.9950 & 0 \end{pmatrix} \quad Bc = \begin{pmatrix} 0.9975 \\ 0 \end{pmatrix}$$
$$Cc = \begin{pmatrix} -1.1347 & 1.0044 \end{pmatrix} \quad Dc = 4.1150$$

Although $\|T_{\zeta v}\|_{\mathcal{H}_\infty} = 1.43$ a simple analysis shows that the closed–loop system is stable for all $0.5 \leq k \leq 2$.

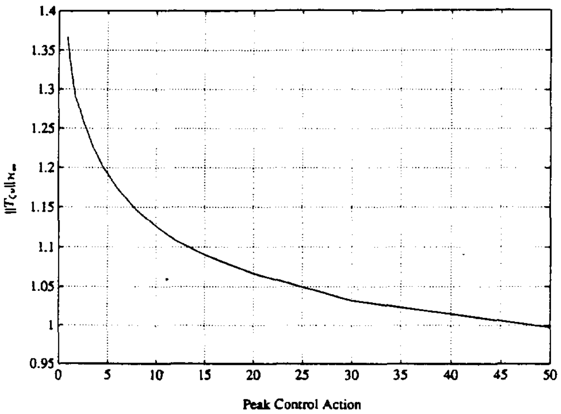

Figure 7. Peak Control Action vs. $\|T_{\zeta v}\|_{\mathcal{H}_\infty}$

5. Conclusions

Most realistic control problems involve both some type of time and frequency domain performance requirements and certain degree of model uncertainty. However, the majority of control design methods currently available focus only on one aspect of the problem.

We recently proposed to address this type of problems using a mixed $l_\infty/\mathcal{H}_\infty$ optimization approach. In this approach, the degrees of freedom available in the problem are used to optimize a time–domain performance measure over all the controllers that guarantee a desired robustness level, expressed in terms of the $\|.\|_{\mathcal{H}_\infty}$ of a transfer function. The resulting convex optimization problem can be decoupled into a finite dimensional, albeit non–differentiable, constrained optimization and an unconstrained Nehari approximation problem. Thus, the solution does not necessitate the use of approximations used in some previous approaches.

The examples of sections III and IV highlight both the strengths and weaknesses of the proposed design paradigm: The method allows for dealing explicitly with time–domain specifications, removing some of the undesirable features of optimal $\mathcal{H}_\infty$ controllers. However, it may result in very large order controllers (twice the number of time elements of $\{\psi_k\}$ considered), necessitating some type of model reduction.

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