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A Receding Horizon State Dependent Riccati Equation Approach to Suboptimal Regulation of Nonlinear Systems[1]

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Abstract

The problem of rendering the origin an asymptotically stable equilibrium point of a nonlinear system while, at the same time, optimizing some measure of performance has been the object of much attention in the past few years. In contrast to the case of linear systems where several optimal synthesis techniques (such as $\mathcal{H}_\infty$, $\mathcal{H}_2$ and ℓ^1) are well established, their nonlinear counterparts are just starting to emerge. Moreover, in most cases these tools lead to partial differential equations that are difficult to solve. In this paper we propose a suboptimal regulator for nonlinear affine systems based upon the combination of receding horizon and state dependent Riccati equation techniques. The main result of the paper shows that this controller is nearly optimal provided that a certain finite horizon problem can be solved on–line. Additional results include sufficient conditions guaranteeing closed loop stability even in cases where there is not enough computational power available to solve this optimization on–line, and an analysis of the suboptimality level of the proposed method.

1 Introduction

A large number of control problems involve designing a controller capable of rendering some point an asymptotically stable equilibrium point of a given time invariant system while simultaneously optimizing some performance index. In the case of nonlinear dynamics, popular design techniques include feedback linearization (FL) [16], the use of control Lyapunov functions (CLF)[1, 10, 21], recursive backstepping [16], and recursive interlacing [18]. While these methods provide powerful tools for designing globally (or semi-globally) stabilizing controllers, performance of the resulting closed loop systems can vary widely. To illustrate this point consider the following system [10]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -e^{x_2}\left(x_1+\tfrac{1}{2}x_2\right)+\tfrac{1}{2}x_2e^{4x_1+3x_2}+e^{2x_1+2x_2}u \end{aligned} \quad (1)$$

with initial condition $[-2 \quad 0]'$. It can be shown that the optimal control law that minimizes the performance index

$$J=\int_0^\infty \left(x_2^2+u^2\right)dt$$

is given by $u^*=-x_2e^{2x_1+x_2}$, with the corresponding value of $J^*=4$. On the other hand, FL and CLF designs yield the values $J_{FL}=238$ and $J_{CLF}=390$ (see [10] for details). Indeed, while the methods mentioned aboved can recover the optimal under certain conditions [8, 6], in general there are no guarantees on the performance of the resulting system.

As an alternative, during the past few years nonlinear counterparts of $\mathcal{H}_\infty$ [2, 11, 14, 20] and ℓ^1 [15] have started to emerge. However, from a practical standpoint they suffer from the fact that they lead to Hamilton–Jacobi–Isaacs type partial-differential equations that are hard to solve, except in some restrictive, low–dimensional cases.

In this paper we propose an alternative controller for suboptimal regulation of non–linear affine systems. This controller is based upon the combination of receding horizon and State Dependent Riccati Equation (SDRE, [4]) ideas, and follows in the spirit of a similar controller successfully used in the case of constrained linear systems [22, 23]. The main result of the paper shows that this controller is nearly optimal and globally stabilizes the plant, provided that enough computational power is available to solve on–line a ***finite horizon*** opti-

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mization problem. In cases where this condition fails, we show how to modify the proposed controller to guarantee stability (possibly at the expense of optimality) and we establish a connection with the well known CLF methodology. Additional results include an analysis of the suboptimality of the proposed method and show that if an approximate solution to the problem is known in a set containing the origin, then our controller yields an extension of this solution with the same suboptimality level.

Due to space constraints, all the proofs have been omitted. A full version of the paper can be obtained by contacting the authors.

2 Preliminaries

2.1 Notation and Definitions

Consider the control–affine nonlinear system:

$$\dot{x} = f(x) + g(x)u \tag{2}$$

where $x \in R^n$, $u \in R^m$, the vector fields $f(.)$ and $g(.)$ are known C^1 functions, and where $f(0) = 0$. Given a function $V\colon R^n \to R$ its Lie derivative along f is defined as

$$L_f V(x) = \frac{\partial V}{\partial x} f(x)$$

Definition 1 *A positive definite and radially unbounded C^1 function $V : R^n \to R_+$ is a Control Lyapunov function for the system (2) if*

$$\inf_u [L_f V(x) + L_g V(x)u] < 0, \qquad \forall\, x \neq 0 \tag{3}$$

It is well known (see for example [16], pag 26) that existence of a control Lyapunov function is equivalent to the existence of a globally asymptotically stabilizing feedback control law $u(x)$[1].

2.2 The Nonlinear Regulator Problem

Consider the nonlinear system (2). Our goal is to find a feedback control law $u(x)$ that minimizes the following performance index

$$J(x_o, u) = \int_0^\infty [x'Q(x)x + u'R(x)u]\, dt,\ x(0) = x_o \tag{4}$$

where $Q(.)$ and $R(.)$ are C^1, positive definite matrices[2]. It is well known [3] that this problem is equivalent to solving the following Hamilton–Jacobi–Bellman partial differential equation:

$$0 = \frac{\partial V}{\partial x} f - \frac{1}{4}\frac{\partial V}{\partial x} g R^{-1} g' \frac{\partial V}{\partial x}' + x'Q(x)x,\ V(0) = 0 \tag{5}$$

If this equation admits a C^1 nonnegative solution V, then the optimal control is given by $u = -\frac{1}{2}R^{-1}g'\frac{\partial V}{\partial x}'$ and $V(x)$ is the corresponding optimal cost (or storage function), i.e.

$$V(x) = \min_u \int_0^\infty (x'Qx + u'Ru)\, dt$$

3 An Equivalent Finite Horizon Regulation Problem

Unfortunately, the complexity of equation (5) prevents its solution except in some very simple, low dimensional cases. In this section we introduce a finite horizon approximation of the nonlinear regulation problem stated in section 2.2 and we analyze its properties. This approximation forms the basis of the proposed method.

Lemma 1 *Consider an compact set S containing the origin in its interior and assume that the optimal storage function $V(x)$ is known for all $x \in S$. Let $c = \min_{x\in\partial S} V(x)$ where ∂S denotes the boundary of S. Finally, define the set $S_v = \{x\colon V(x) < c\}$. Consider the following two optimization problems:*

$$min_u \left\{ J(x,u) = \int_0^\infty [x'Q(x)x + u'Ru]\, dt \right\} \tag{6}$$

$$min_u \left\{ J_T(x,u) = \int_0^T (x'Q(x)x + u'Ru)\, dt + V[x(T)] \right\} \tag{7}$$

subject to (2). Then the following facts hold:

1. *An optimal solution of problem (7) is also optimal for (6) in the interval $[0,T]$ provided that $x(T) \in S_v$.*
2. *Consider now $T_1 > T$. If $x(T) \in S_v$ then a controller that optimizes J_T is also optimal with respect to J_{T_1} in $[0,T]$.*

This Lemma shows that if a solution to the HJB equation (5) is known in a neighborhood of the origin, then it can be extended via an explicit finite horizon optimization, well suited for an on–line implementation. This suggest a receding horizon type control combining an on–line optimization with an

[1] If in addition the so–called small control property holds then the stabilizing control law is continuous.

[2] This condition can be relaxed to $Q(x) \geq 0$

off-line phase to find a local solution to (5). However, finding (and storing) this local solution can be very computationally demanding in cases where the dimension of the problem is not low. Thus it is of interest to consider the case where an approximation $\Psi(x)$ rather than the true storage function is used in (7). The next result shows that in this case the approximation error *does not grow* (to the first order). In other words, the difference between the true optimal $V[x(0)]$ and $J_T[x(0)]$ is approximately equal to the difference between $V[x(T)]$ and $\Psi[x(T)]$.

Theorem 1 *Let $\Psi: R^n \rightarrow R_+$ be a positive definite function and consider the following optimization problem:*

$$J_\Psi(x,t) = \min_u \int_t^T (x'Qx + u'Ru)\, d\tau + \Psi[x(T)] \tag{8}$$

subject to (2). Define the approximation error $e(x,t) \doteq J_\Psi(x,t) - V(x)$ and assume that Ψ is selected so that $\frac{\partial e[x(T),T]}{\partial x} \cong 0$. Then $J_\Psi(x,t) - V[x(t)] \cong \Psi[x(T)] - V[x(T)]$ (to the first order).

4 Proposed Control Algorithm

From Lemma 1 it follows that, given an initial condition $x(0)$, problem (6) can be solved by solving a sequence of problems of the form (7) with increasing T until a solution such that $x(T) \in S_v$. Moreover, once such a solution is obtained, no further improvement of the cost can be achieved by increasing the horizon T. These results suggest the following receding–horizon type control law. Let $x(t)$ denote the current state of system (2). Then:

1. If $x(t) \in S_v$, $u = -\frac{1}{2}R^{-1}g'\frac{\partial V}{\partial x}'$
2. If $x(t) \notin S_v$ then solve a sequence of optimization problems of the form (7) until a solution such that $x(T) \in S_v$ is found. Use the corresponding control law $u(t)$ in the interval $[t_o, t_o + \delta t]$.

From the results above it is clear that the resulting control law is globally optimal and thus globally stabilizing. However, as we indicated before, the computational complexity associated with finding $V(x)$ (even only in the region S_v) may preclude the use of this control law in many practical cases. Thus, it is of interest to consider a modified control law where an approximation $\Psi(x)$ (rather than $V(x)$) is used. To this effect consider a compact set S containing the origin in its interior and let $\Psi: S \rightarrow R_+$, $\Psi \in \mathcal{C}^1$ be a Control Lyapunov Function for system (2). Denote by u_ψ the corresponding control law. Finally, let $c = \min_{x \in \partial S} \Psi(x)$ and define the set $S_\Psi \subseteq S = \{x: \Psi(x) \leq c\}$. Then we propose the following modified control law:

1. If $x \in S_\psi$, $u_\psi(x) \doteq \underset{u}{\operatorname{argmin}}\{L_f\Psi + L_g\Psi u\}$
2. If $x \notin S_\Psi$ then consider an increasing sequence T_i. Let

   $$u_T^* = \operatorname{argmin}\left\{\int_o^T (x'Qx + u'Ru)\, dt + \Psi[x(T)]\right\}$$

   Denote by $x^*(.)$ the corresponding optimal trajectory and define: $T(x) = \inf\{T : x^*(T) \in S_\Psi\}$[3]. Then $u_\psi(x) \doteq u^*_{T(x)}(t)$, $t \in [t_o, t_o + \delta t]$.

Note that from Theorem 1 it follows that the sub-optimality associated with the modified algorithm is approximately given by $e_\Psi = \sup_{x \in S_\Psi} |\Psi(x) - V(x)|$.

Theorem 2 *Assume that $\underline{\sigma}[Q(x)] > \sigma_m > 0$ for all x, where $\underline{\sigma}(.)$ denotes the minimum singular value. Then the control law u_ψ globally stabilizes (2)*

5 A Modified Receding Horizon Controller

In the last section we outlined a receding horizon type law, that under certain conditions, is nearly optimal and globally stabilizes system (2). While most of these conditions are rather mild (essentially equivalent to the existence of a CLF), the requirement that T should be large enough so that $x(T) \in S_\psi$ could pose a problem, specially in cases where the system has fast dynamics. In this section we propose a modified control law that is guaranteed to stabilize the system, even when this condition fails, and that takes into account computational time constraints.

Consider the following receding horizon control law:

Algorithm 1

0.- Data: a CLF $\Psi(x)$, an invariant region S_Ψ such that $0 \in int\{S_\Psi\}$, a horizon T.

[3] From Barbalat's Lemma ([16], pag. 491) we have that $x'Qx + u'Ru \rightarrow 0$ as $T \rightarrow \infty$. Hence for every x_o, $T(x_o)$ is finite.

1.- If $x(t) \in S_\psi$, $u_\psi(x) = \underset{u}{\operatorname{argmin}} \{L_f\Psi + L_g\Psi u\}$

2.- If $x(t) \notin S_\Psi$ then

$$u_\Psi = \underset{u}{\operatorname{argmin}} \left\{ \int_t^{T+t} \left(x'Qx + u'Ru\right) dt + \Psi\left[x(T+t)\right] \right\} \quad (9)$$

subject to:

$$\begin{aligned} &-\sigma[x(t+T)] > x(t+T)'Qx(t+T) + L_f\Psi\Big|_{x(t+T)} \\ &+\min_u \left\{ u'(t+T)Ru(t+T) + L_g\Psi\Big|_{x(t+T)} u \right\} \\ &-x(t)'Qx(t) - u^{*\prime}(t)Ru^*(t) \end{aligned} \quad (10)$$

where $\sigma(x)$ is some positive definite function such that $\sigma(x) \leq x'Qx$.

Theorem 3 *The control law u_Ψ generated by Algorithm 1 has the following properties:*

1. *It renders the origin a globally asymptotically stable equilibrium point of (2)*
2. *Coincides with the globally optimal control law when $\Psi(x) = V(x)$.*
3. *Is nearly optimal (in the sense of Theorem 1) when $x(T) \in S_\Psi$.*

Remark 1 *Let $\Psi(x) = x'P(x)x$, where P denotes the solution to the SDRE*

$$\begin{aligned} &A'(x)P(x) + P(x)A(x) \\ &-P(x)B(x)R^{-1}(x)B'(x)P(x) + Q(x) = 0 \end{aligned} \quad (11)$$

where $f(x) = A(x)x$ and $B(x) = g(x)$. It can be shown that there exists T_o such that for all $T > T_o$ the constraints (10) are feasible. It follows that $\Psi(x) = x'P(x)x$ is a suitable choice for the terminal penalty. Moreover from the properties of the SDRE method (see [4]) it follows that with this choice, the control law satisfies all the necessary conditions for optimality as $\mathcal{O}\left[||x(t+T)||^2\right]$

Finally, before closing this section we consider a modified control law that takes into account the sample and hold nature of receding horizon implementations.

Algorithm 2

0.- Data: a CLF $\Psi(x)$, an invariant region S_Ψ such that $0 \in int\{S_\Psi\}$, a horizon T, a sampling interval T_s.

1.- If $x(t) \in S_\psi$, $u_s(x) = \underset{u}{\operatorname{argmin}} \{L_f\Psi + L_g\Psi u\}$

2.- If $x(t) \notin S_\Psi$ then

$$u_s = \underset{u}{\operatorname{argmin}} \left\{ \int_t^{T+t} \left(x'Qx + u'Ru\right) dt + \Psi\left[x(T+t)\right] \right\} \quad (12)$$

subject to:

$$\begin{aligned} - \quad & \sigma[x(\tau+t+T)] \\ > \quad & x(\tau+t+T)'Qx(\tau+t+T) + L_f\Psi\Big|_{x(\tau+t+T)} \\ + \quad & \min_u \{u'(\tau+t+T)Ru(\tau+t+T) \\ + \quad & L_g\Psi\Big|_{x(\tau+t+T)} u \Big\} \\ - \quad & x(t)'Qx(t) - u^{*\prime}(t)Ru^*(t), \\ & \text{for all } 0 \leq \tau \leq T_s \end{aligned} \quad (13)$$

Lemma 2 *The control law u_s renders the origin a globally stable equilibrium point of (2). Moreover, it is nearly optimal and coincides with the globally optimal control law when $\Psi(x) = V(x)$.*

6 Illustrative Example

Example 1 *This example consists of a planar ducted fan (a simplified model of a thrust vectored aircraft). The dynamics are given by (see [17, 8] for details)*

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -g\sin\theta \\ g\left(\cos\theta - 1\right) \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\cos\theta}{m} & -\frac{\sin\theta}{m} \\ \frac{\sin\theta}{m} & \frac{\cos\theta}{m} \\ \frac{r}{J} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (14)$$

where x, y and θ denote horizontal, vertical and angular position respectively and where u_1 and u_2 are the control inputs. The numerical values of the parameters are $m = 4Kg$, $J = 0.0475Kgm^2$ and $r = 0.26m$. The goal is to minimize a performance index of the form (4) with:

$$Q = diag\,[5 \quad 5 \quad 1 \quad 1 \quad 1 \quad 5],\; R = I_{2\times 2}$$

corresponding to the following choice of state variables: $\xi = [x \quad y \quad \theta \quad \dot{x} \quad \dot{y} \quad \dot{\theta}]$.

Table 1 shows the result corresponding to the initial condition $\xi(0) = [0 \quad 0 \quad 0 \quad 12.5 \quad 0 \quad 0]$. Note that in this case the SDRE solution yields the second lowest cost.

Figure 1 shows a comparison of the trajectories generated by the SDRE and RSDRE method, with

method	V
LQR [8]	1.1×10^5
CLF [8]	2.53×10^4
LPV [8]	1833
SDRE	1989
RSDRE	1140

Table 1: Comparison of different methods for the DFAN example

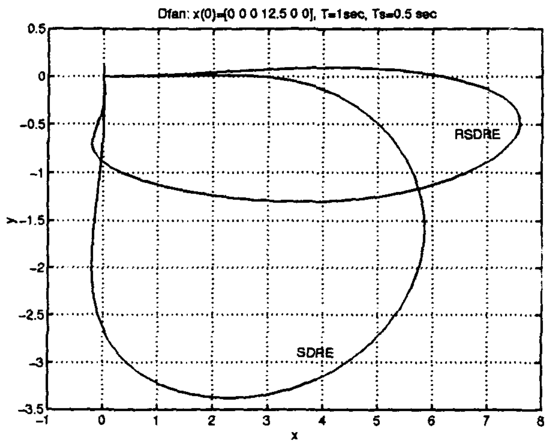

Figure 1: SDRE and RSDRE trajectories for the Dfan example

$T = 1sec$ and $T_s = 0.5sec$. In this case the RSDRE method produced a cost virtually equal to the global optimal (found off-line by numerical optimization). This can be explained by looking at the plots in Figure 2. These plots show that while $\Psi(x) = x'P(x)x$ gives initially a very poor estimate of the cost-to-go, the combination of $\Phi(x)$ and the explicit integral in (9) give a very good estimate if T is chosen $\geq 1sec$. It is worth mentioning that a conventional receding horizon controller (i.e. one obtained by setting $\Psi \equiv 0$ in (9)) with the same choice of horizon and sampling time fails to stabilize the system.

7 Conclusions

In contrast with the case of linear plants, tools for simultaneously addressing performance and stability of nonlinear systems have emerged relatively recently. Recent counterexamples [7, 8] illustrated the fact that while several commonly

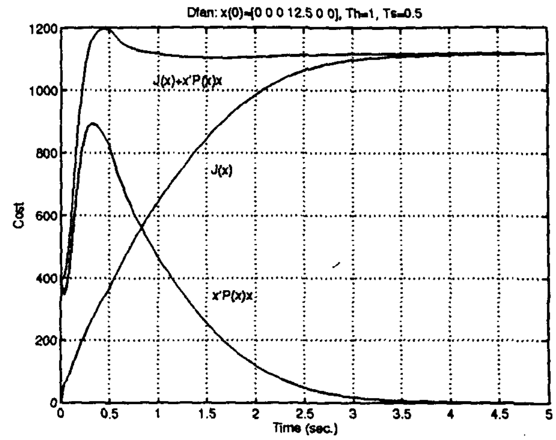

Figure 2: The terms of the cost as function of the horizon

used techniques can successfully stabilize nonlinear systems, the resulting closed-loop performance varies widely. Moreover, these performance differences are problem dependent, with performance of a given method ranging from (near) optimal to very poor.

In this paper we propose a new suboptimal regulator for affine nonlinear systems, based upon the combination of receding horizon and state dependent Riccati equation techniques. The main result of the paper shows that under certain relatively mild conditions this regulator renders the origin a globally asymptotically stable equilibrium point. In the limit as the horizon $T \rightarrow 0$, the proposed control law reduces to the inverse optimal controller proposed by Freeman and Kokotovic [9]. Thus, these conditions are essentially equivalent to the existence of a Control Lyapunov Function. Additional results in the paper show that the regulator is near optimal, provided that a good approximation to the optimal storage function is known in a neighborhood of the origin. These results were illustrated with a practical example where the proposed controller outperformed several other commonly used techniques. Finally, note in passing that the finite approximation (8) is also valuable as a tool to speed-up off line numerical computation of near optimal solutions, for instance when combined with conjugate gradient type algorithms [5].

An issue that was not addressed in this paper is that of the computational complexity associated with solving the nonlinear optimization problem (9). Following [8] this complexity could be reduced by exploiting differential flatness to perform the optimization in flat space. Additional research being pursued includes the explicit incorporation of state and control constraints into the formalism and its extension to handle model uncertainty.

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