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A superstability approach to synthesizing low order suboptimal $\mathcal{L}^\infty$–induced controllers for LPV systems

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Abstract

In this paper we illustrate how the newly introduced concept of superstability can be used to synthesize low–complexity controllers for LPV systems, with guaranteed $\mathcal{L}_\infty$ disturbance rejection properties. As shown here, in the superstability context this problem admits a simple solution, both in the continuous and discrete time cases. Moreover, in the case of parameter–affine dynamics, it leads to simple gain–scheduled controllers.

1 Introduction

A widely used idea to handle non–linear dynamics is to linearize the plant around several operating points and then use *linear* control tools to design a controller for each of these points. The actual controller is implemented using gain scheduling. However, while intuitively appealing, this idea has several pitfalls [11]. Motivated by these shortcomings, considerably attention has been devoted to the problem of synthesizing controllers for Linear Parameter Varying Systems, where the state–space matrices of the plant depend on time–varying parameters whose values are not known a priori, but can be measured by the controller. Assuming that bounds on both the parameter values and their rate of change are known, then Affine Matrix Inequalities based conditions are available guaranteeing exponential stability of the system. Moreover, these conditions can be easily used to synthesize stabilizing controllers guaranteeing worst case performance bounds (for instance in an $\mathcal{H}_2$ or $\mathcal{H}_\infty$ sense [1, 7, 3, 12]). On the other hand, the $\mathcal{L}_\infty$–induced performance case is considerably less developed. Most of the work dealing with LPV systems in an $\mathcal{L}_\infty$ setup deals with synthesizing robust non–linear controllers that guarantee some robust performance level for the entire family of plants, without exploiting information about the present value of the parameter [6].

In this paper we consider the problem of rejection of persistent $\mathcal{L}_\infty$ bounded disturbances, both for continuous and discrete time systems. Our main result shown that in these cases *superstability* [10] can be exploited to synthesize low–complexity LPV controllers with guaranteed $\mathcal{L}_\infty$ induced closed–loop performance. Moreover, in the case where the dynamics of the plant depends affinely on the parameters, these controllers reduce to simple gain–scheduling. At the present time, the main drawback of the approach stems from the fact that, since superstability is a stronger property than stability, not every controllable system can be rendered superstable with a simple controller. While the general case of MIMO systems is still open, as we show in the paper, this issue can be addressed, at the price of a more involved controller, in the SISO case.

2 Notation

In the sequel, we use $\|\cdot\|$ to denote the ∞-norm for vectors: $\|a\| \doteq \max_i |a_i|$, $\quad a = (a_1, \ldots, a_n)^T \in \mathcal{R}^n$, and $\|\cdot\|_\infty$ the ℓ^∞ ($\mathcal{L}^\infty$) norm of a sequence (function). Also, we use the 1-norm for matrices: $\|A\|_1 \doteq \max_i \sum_j |a_{ij}|$, $A = (a_{ij})$ and for polynomials: for $b(z) = b_0 + b_1 z + \ldots + b_n z^n$, $\|b\|_1 = \sum_i |b_i|$. Finally, given a matrix A, we will denote its i^{th} row by $A(i,:)$.

Definition 1 *A matrix $A = ((a_{ij}))$ is said to be d-superstable, if it satisfies the following condition:*

$$\mu_d(A) = \mu_d \doteq 1 - \|A\|_1 > 0. \qquad (1)$$

Similarly, a matrix A is said to be c-superstable if it satisfies the condition

$$\mu_c(A) = \mu_c \doteq \min_i \Big(-a_{ii} - \sum_{j \neq i} |a_{ij}| \Big) > 0. \quad (2)$$

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Clearly, a *superstable* matrix is stable, but the converse does not hold. In the sequel, we refer to matrices satisfying either (1) or (2) simply as *superstable* and use μ to denote either μ_d or μ_c when the meaning is clear from the context.

3 Stability and Disturbance Rejection Properties of LPV Superstable Systems

In this section we show that, as in the LTI case, superstability of an LPV system implies both exponential stability of the origin, and $\mathcal{L}_\infty$ stability. In the sequel we will consider LPV systems of the form:

$$\sigma x = A[\rho(t)]x(t) + B[\rho(t)]w(t) \qquad (3)$$

where σ denotes either the one step–advance (discrete time) or time–derivative (continuous time) operator, $x \in R^{n_x}$, and $w \in R^{n_w}$ represent the state and exogenous disturbances, respectively, $\rho \in R^{n_\rho}$ denotes a vector of time–varying parameters and where the matrix functions $A(.)$ and $B(.)$ are continuous. Further, we will assume that at all times $\rho(t) \in \mathcal{P} \subset R^{n_\rho}$, where $\mathcal{P}$ is a given compact set.

Definition 2 *The system (3) is said to be superstable if $A[\rho(t)]$ is superstable for all $\rho \in \mathcal{P}$.*

Lemma 1 *If the LPV system (3) is superstable, then the following facts hold for all admissible parameter trajectories $\rho(.)$:*

a) *If $w_k \equiv 0$ for all $k \geq 0$ then*

$$\|x(k)\| \leq (1-\mu)^k \|x_0\| \qquad (4)$$

where $\mu = \inf_{\rho \in \mathcal{P}} \mu_d\{A[\rho]\}$ in the discrete-time case and

$$\|x(t)\| \leq e^{-\mu t}\|x(0)\| \qquad (5)$$

for the continuous-time counterpart, where $\mu = \inf_{\rho \in \mathcal{P}} \mu_c\{A[\rho]\}$

b) *Let $\mu_i(\rho) = 1 - \sum_i |a_{ij}(\rho)|$ in the discrete time case and $\mu_i(\rho) = -a_{ii}(\rho) - \sum_{j \neq i} |a_{ij}(\rho)|$ in the continuous time case. If $\|w\|_\infty \leq 1$ and $\|x_0\| \leq \gamma \doteq \max_{\rho \in \mathcal{P}} \max_i \frac{\|B(i,:)\|_1}{\mu_i}$ then*

$$\|x\|_\infty \leq \gamma \qquad (6)$$

in both cases.

Proof: In the discrete time case, property a) follows from the fact that:

$$\begin{aligned} \|x(k+1)\|_\infty &\leq \textstyle\prod_{j=0}^{k} \|A[\rho(j)]\|_1 \|x_o\|_\infty \\ &\leq (1-\mu)^{(k+1)}\|x_o\|_\infty \end{aligned}$$

To prove the continuous time counterpart, define $v(t) = \max_i x_i(t)^2$. It can be easily shown that the upper right–hand derivative of v, D^+v satisfies:

$$D^+ v \leq -2\mu\{A[\rho(t)]\}\, v \qquad (7)$$

Hence, from the Comparison Principle ([8], page 651) it follows that

$$\begin{aligned} v(t) &\leq e^{-2\int_o^t \mu(t)dt} \\ &\Rightarrow \\ \|x(t)\|_\infty &\leq e^{-\mu t}\|x(0)\|_\infty \end{aligned} \qquad (8)$$

We will prove property (b) by establishing that the hypercube $\{x \in \mathcal{R}^n : \|x\| \leq \gamma\}$ is d–invariant [4]. To this effect, note that in the discrete time case we have that

$$\begin{aligned} |x(k+1)_i| \leq & \textstyle\sum_{i=1}^{n} |a_{ij}||x(k)_j| + \\ & \textstyle\sum_{i=1}^{n_w} |b_{ij}| \leq \gamma \end{aligned} \qquad (9)$$

Similarly, for continuous time systems, consider a point on the boundary of the hypercube, and assume withouth loss of generality that $|x_i(t)| = \gamma$, $i = 1, \ldots, m$ and $|x_i(t)| < \gamma$, $i = m+1, \ldots, n$. Then for $i = 1, \ldots, m$ we have that:

$$\begin{aligned} &\tfrac{1}{2}\tfrac{d}{dt}x_i^2 = a_{ii}x_i^2 \\ &+ \textstyle\sum_{i \neq j} a_{ij}x_i x_j + x_i \sum_j b_{ij}w_j \\ &\leq (a_{ii} + \textstyle\sum_{i \neq j}|a_{ij}|)\gamma^2 + \sum_j |b_{ij}||w_j|\gamma \\ &\leq (-\mu + \tfrac{\|B\|_1}{\gamma})\gamma^2 < 0 \end{aligned} \qquad (10)$$

Remark 1 *Note that there is no counterpart of these results if superstability is relaxed to just stability: it is well known that frozen–time stability of the matrix $A(t)$ does not guarantee stability of the origin (see for instance example 3.22 in [8]).*

Next, we exploit these results to synthesize simple controllers with guaranteed performance properties. Consider now an LPV system of the form system

$$\begin{aligned} \sigma x(t) &= A(\rho)x(t) + B_1(\rho)w(t) + B_2(\rho)u(t) \\ y(t) &= x + D(\rho)w \end{aligned} \qquad (11)$$

where u and y denote the control action and the outputs available to the controller, respectively. From Lemma 1 it follows that if there exists a matrix $K(\rho)$ such that the corresponding (frozen–time) closed loop matrix $A_{cl} = A[\rho(t)] + B_2[\rho(t)]K(\rho)$ is superstable, then the control action $u = K(\rho)x$ renders the origin an exponentially stable equilibrium point of the closed–loop system. As pointed out in [10], existence of a matrix K that superstabilizes a given pair (A, B), while not guaranteed, reduces to a LP feasibility problem. This property allows for recasting the problem of finding a super-stabilizing LPV controller into the following *parametric* (in ρ) LP form, for both continuous and discrete–time systems):

$$\max_{K} \mu\left[A(\rho) + B_2(\rho)K\right] \tag{12}$$

If this problem admits a solution $\hat{\mu}(\rho) > 0$ for all $\rho \in \mathcal{P}$, then the additional degrees of freedom available in the problem can be exploited to optimize performance, for instance measured in terms of the $\mathcal{L}_\infty$ induced norm.

4 Persistent Disturbance Rejection

The goal here is to design a controller such that the worst case $\mathcal{L}_\infty$ norm over all feasible parameter trajectories and persistent bounded disturbances $\|w\| \leq 1$ is minimized, that is

$$\min_{u} \sup_{\rho\in\mathcal{P}, \|w\|_\infty \leq 1} \|x\|_\infty \tag{13}$$

The problem above can be solved by finding the maximal controlled–invariant set and the associated non–linear controller [6]. However, the complexity of this controller can be arbitrarily high. On the other hand no known solutions to this problem exist when the controller is restricted to the class of LPV controllers with bounded complexity[1]. To circumvent this difficulty, we propose to minimize, rather than $\|x\|_\infty$, an upper bound motivated by (6). This leads to the following problem:

Problem 1 *Find a superstabilizing LPV controller that minimizes the upper bound (6) over all parameter trajectories, that is:*

$$J^* \doteq \max_{\rho\in\mathcal{P}} \min_{K(\rho),\nu} \left\{ \max_{i} \|(B_1(\rho) + B_2(\rho)K(\rho)D(\rho))(i,:)\|_1/\nu_i \right\}$$

subject to

$$\mu_i\left[A(\rho) + B_2(\rho)K(\rho)\right] \geq \nu_i > 0, \ \forall \rho \in \mathcal{P}$$

Since the constraints are linear in the entries of the matrix $K(\rho)$, this problem can be recast as a Linear Programming problem, parametric in ρ and ν. The resulting controller has guaranteed closed–loop performance, in the sense that

$$\sup_{\rho\in\mathcal{P}, \|w\|_\infty \leq 1} \|x[t, \rho(t)]\|_\infty \leq J^* \tag{14}$$

Since an upper bound of the state vector is minimized uniformly over time rather than just asymptotically, this approach prevents "peaking effects," in the initial part of the trajectory.

5 Implementation Consideration

As shown above, in the context of superstability, the disturbance rejection problems lead to parametric LP problems. In simple cases, these problems may admit an explicit solution. Alternatively, if the dimension of the plant is not too large, these problems can be solved on–line. However, in many cases of practical interest, neither approach may be feasible. In the case of general dependence of the plant on the time–varying parameters ρ this may require griding the parameter space, solving the parametric LP problems at each point in the grid and interpolating the resulting controllers. This is precisely the approach pursued in [12] to deal with the $\mathcal{H}_\infty$ case. However, as we show in the sequel, in the case of *affine* dependence on the parameters, superstability leads to a simple gain–scheduled controller, avoiding the need for griding and interpolating.

Theorem 1 *Assume that $A(\rho) = \sum_{i=1}^{m} \rho_i A_i$, $\rho_i > 0$, $\sum \rho_i = 1$ and that there exists K_i such that $A_i + B_2K_i$, $i = 1, \ldots, n_\rho$ is superstable. Then the solution to Problem 1 is given by*

$$K(\rho) = \sum_i \rho_i K_i \tag{15}$$

where the matrices K_i solve:

$$\min_{K_i} \sup_j \frac{\|(B_1 + B_2K_iD)(j,:)\|_1}{\mu[(A_i + B_2K_i)(j,:)]} \tag{16}$$

Proof From (1) and (21) it can be easily shown that since the matrices $A_i + B_2K_i$ are superstable, the controller $K(\rho) = \sum_i \rho_i K_i$ renders the LPV system $A(\rho) + B_2K(\rho)$ superstable. To show optimality, begin by noticing that the hypercube $\mathcal{H}_J \doteq$

[1] In contrast, when performance is measured in the ℓ_2 induced sense and the states are available for feedback, the problem admits a solution of the form $u = K(\rho)x$.

$\{x \in R^n : \|x\|_\infty \leq J^*\}$ is the smallest hypercube that can be rendered invariant for all $\rho \in \mathcal{P}$ by an LPV superstabilizing controller. Thus, to show optimality, we only need to show that the controller (15) also renders $\mathcal{H}$ invariant. In the case of discrete–time systems we have that, if $x(t) \in \mathcal{H}$ then:

$$\begin{aligned} &|x(t+1)_r| = \\ &\Big|\sum_{i=1}^{n_\rho} \rho_i \sum_s (A_i + B_2 K_i)_{rs} x(t)_s \\ &+ \sum \rho_i \sum_s (B_1 + B_2 K_i D)_{rs} w(t)_s\Big| \\ &\leq \sum_{i=1}^{n_\rho} \rho_i (\sum_s |(A_i + B_2 K_i)_{rs}| J^* + \\ &\|B_1^{i,cl}(r,:)\|_1) \\ &\leq J^* \end{aligned} \tag{17}$$

where $B_1^{i,cl} = B_1 + B_2 K_i D$. Similarly, in the continuous time case, given a point $x(t)$ on the boundary of $\mathcal{H}$, $x(t) \in \partial\mathcal{H}$, $|x_i(t)| = J^*$, $i = 1, \ldots, m$, $|x_i(t)| < J^*$; $i = m+1, \ldots, n$ we have that:

$$\begin{aligned} &\tfrac{1}{2}\tfrac{d}{dt} x_i^2 \leq \sum_j \rho_j a_{ii}^{j,cl} x_i^2 \\ &+ \sum_{i \neq k} |\sum_j \rho_j a_{ik}^{j,cl}| J^2 + |x_i| \sum_k |b_{ik}^{cl}| |w_k| J^* \\ &\leq J^{*2} \sum_j \rho^j \left(a_{ii}^{j,cl} + \sum_k |a_{ik}^{j,cl}| + \tfrac{\|B^{j,cl}(i,:)\|_1}{J^*} \right) \\ &\leq 0 \end{aligned} \tag{18}$$

where $a_{rs}^{j,cl}$ denotes the r, s element of $A_{j,cl} \doteq A_j + B_2 K_j$ and $B^{j,cl} = B_1 + B_2 K_j D$.

6 A Simple Example

Consider a discrete–time affine LPV system of the form described in Theorem 1, having three vertex matrices

$$A_1 = \begin{pmatrix} -0.96 & -0.24 \\ 0.36 & 0.66 \end{pmatrix}; \; A_2 = \begin{pmatrix} 0.00 & -0.14 \\ 0.42 & -0.40 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -0.62 & 0.36 \\ -0.62 & -0.40 \end{pmatrix}$$

constant input matrices

$$B_1 = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \quad B_2 = \begin{pmatrix} 1.0 \\ -2.3 \end{pmatrix}$$

and D=0. In accordance with Theorem 1, to find the optimal control, we first determine the feedback gains K_1, K_2 and K_3. Finding this matrices reduces to solving three LPs (as mentioned in Theorem 1). The gain matrices obtained are

$$K_1 \approx \begin{pmatrix} 0.3857 \\ 0.2584 \end{pmatrix}^T; \quad K_2 \approx \begin{pmatrix} 0.1178 \\ -0.0883 \end{pmatrix}^T$$

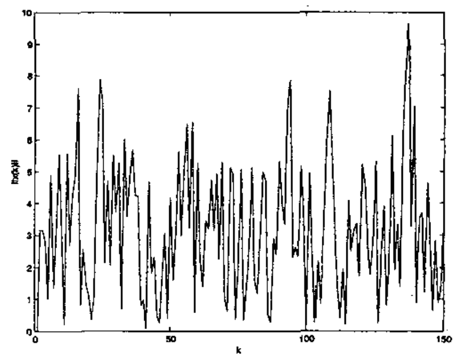

Figure 1: Norm of $x(k)$ versus time instant k

$$K_3 \approx \begin{pmatrix} -0.0402 \\ -0.2706 \end{pmatrix}^T.$$

The optimal control law is

$$u = K(\rho)x = \sum_{i=1}^{3} \rho_i K(i) x$$

leading to $\mathcal{L}_\infty$ upper bound.

$$J^* \approx 11.9855$$

To test the controller above, we generated random sequences $\rho(k)$ and $w(k)$ and simulated the above closed loop system. A typical trajectory of $\|x(k)\|$ with $x(0) = 0$ is shown in Figure 1. As it can be seen from the plot, the $\|x\|_\infty$ is below the upper bound J^*, confirming the results presented in the previous section.

7 Dynamic Output Feedback and Some Open Problems

To conclude the paper, in this section we briefly comment on the dynamic output feedback case and on some open problems. As mentioned before, one of the main drawbacks of the superstability approach is that not every stabilizable pair (A, B_2) is superstabilizable. As we show next, in the SISO discrete–time case this difficulty can be solved by extending the concept of superstability to rational transfer functions and using dynamic output feedback.

Definition 3 *A polynomial of the form* $P(z) = 1 + a_1 z + \ldots + a_n z^n$ *is said to be* superstable *if its coefficients satisfy the condition* $\sum_i |a_i| < 1$.

Such polynomials were introduced by Cohn in 1922 and were used by [5] and [9] to synthesize low order suboptimal controllers for LTI systems.

Definition 4 *An LPV SISO system described by the nth order difference equation:*

$$\begin{aligned} x(k) &= a_1(\rho)x(k-1) + \ldots + a_n(\rho)x(k-n) + \\ & \quad b_0(\rho)w(k) + \ldots + b_m(\rho)w(k-m) \end{aligned} \tag{19}$$

is superstable if the characteristic polynomial $d(z,\rho) = 1 - a_1 z - \ldots a_n z^n$ *is superstable for all* $\rho \in \mathcal{P}$.

It can be easily shown that results similar to Lemma 1 hold for a SISO LPV system if its associated polynomial is superstable (the proofs are omitted due to space constraints):

Fact 1 *Consider a superstable discrete–time SISO system of the form (19). Let* $\mu_\rho = 1 - \sum |a_i(\rho)|$ *and* $\mu = \min_{\rho \in \mathcal{P}} \mu_\rho > 0$. *Then, the following properties hold:*

a) If $w_k \equiv 0$ *for all* $k \geq 0$ *then*

$$\|x(k)\| \leq (1-\mu) \max_{1 \leq i \leq n} \|x(k-i)\| \tag{20}$$

b) If $\|w\|_\infty \leq 1$ *and* $\|x_0\| \leq \gamma \doteq \sup_{\rho \in \mathcal{P}} \frac{\|b(\rho)\|_1}{\mu_\rho}$ *then*

$$\|x\|_\infty \leq \gamma \tag{21}$$

This property allows for reducing the problem of synthesizing output feedback LPV controllers to a parametric convex optimization problem as follows: Consider a SISO plant of the form:

$$\begin{bmatrix} e \\ y \end{bmatrix} = \frac{1}{d(z,\rho)} \left[\begin{array}{c|c} n_{11}(z,\rho) & n_{12}(z,\rho) \\ n_{21}(z,\rho) & n_{22}(z,\rho) \end{array} \right] \begin{bmatrix} w \\ u \end{bmatrix} \tag{22}$$

where the scalar signals u, w, y and e represent the control input, exogenous disturbances, measurements available to the controller and performance output respectively, and where z denotes the unit delay operator. In this context, the problem of designing a super–stabilizing LPV controller that optimizes the upper-bound (21) becomes:

Problem 2 *Given the LPV plant (22), find an LPV controller such that*

$$\gamma \doteq \sup_{\rho \in \mathcal{P}} \frac{\|b_{cl}(\rho)\|_1}{\mu_\rho}$$

is minimized

In the sequel we show that this problem reduces to a finite–dimensional convex optimization problem. To this effect consider a controller of the form:

$$c(z,\rho) = \frac{q(z,\rho)}{p(z,\rho)} \tag{23}$$

where p is a monic polynomial of degree n_c. The corresponding closed–loop system is:

$$e = \left[\frac{n_{11}}{d} + \frac{1}{d} \frac{n_{12} q n_{21}}{(dp - n_{22} q)} \right] w. \tag{24}$$

Since the polynomial $[n_{11}n_{22} - n_{12}n_{21}]$ has d as a factor, i.e.

$$n_{11}n_{22} - n_{12}n_{21} = d\bar{n}$$

it follows that

$$(dp - n_{22}q)e = (pn_{11} - q\bar{n})w \tag{25}$$

This last expression can be rewritten as:

$$d_{cl}(p,q)(z,\rho)e = n_{cl}(p,q)(z,\rho)w. \tag{26}$$

Without loss of generality (by using an appropriate scaling if necessary), $p(z,\rho)$ and $q(z,\rho)$ can always be selected such that the polynomial $d_{cl}(p,q)(z,\rho)$ has its independent term equal to one, that is

$$d_{cl}(p,q)(z,\rho) = 1 + d_{cl,1}(\rho)z + d_{cl,2}(\rho)z^2 + \ldots. \tag{27}$$

Thus, Problem 2 can be recast into the following form:

$$\begin{gathered} \min_{p,q} \|n_{cl}\|_1/\gamma \\ \text{subject to} \\ 1 - \|1 - d_{cl}\|_1 \leq \gamma \end{gathered} \tag{28}$$

Since, for a given ρ, $n_{cl}(p,q)$ and $d_{cl}(p,q)$ are affine functions of the coefficients of the polynomials $p(z,\rho)$ and $q(z,\rho)$, and since the additional constraint (27) is equivalent to a linear constraint involving only the leading coefficients of q and p, it follows that solving (28) is equivalent to an LP problem, parametric in γ and ρ.

Remark 2 *In contrast with the MIMO case, notice that Problem 2 always has a solution, provided that the order of the controller is chosen to be at least as large as the order of the plant. This follows from the fact that in this case p and q can be chosen so that, for each ρ, the corresponding closed–loop is a FIR and thus (28) is guaranteed to be feasible for some γ large enough (see [6] for details).*

The results above show that superstabilizing output–feedback dynamic LPV controllers always exists for controllable SISO discrete time systems. On the other hand, it is worth noticing that the standard conversion of a SISO system in the form (19), e.g. an nth order difference equation, to the state space form (3) does not yield a superstable matrix. This suggest that the results presented here for MIMO systems can be expanded to larger classes of systems and that in some cases (at least SISO) superstability is a structural property, independent of the coordinate system chosen for the state-space realization. Finally, the issue of SISO analogs of superstability remains unsolved for continuous-time systems. At the present time no meaningful counterpart of the discrete time superstability concept is known for continuous-time polynomials.

8 Conclusions

In contrast with the case of linear plants, tools for simultaneously addressing performance and stability of linear parameter varying systems have emerged relatively recently, and the theory is far from complete. In particular, few tools are available when its is desired to optimize the $\mathcal{L}_\infty$–induced norm of the closed–loop system.

In this paper, motivated by some earlier results on $\mathcal{L}_\infty$ gain minimization of LTI systems [6, 9, 10], we propose a new suboptimal $\mathcal{L}_\infty$–induced controller for LPV systems. This controller is based upon minimizing an upper bound obtained by imposing *superstability* of the closed–loop system. As shown here, this leads to simple controllers that can be found using parametric Linear Programming. Moreover, in the case of affine dependence of the dynamics with the time–varying parameters, the approach leads to simple gain–scheduled controllers, avoiding the need for griding the parameter space and interpolating the resulting controllers.

At this point, the main drawback of the proposed approach stems from the fact that not every stabilizable system is super–stabilizable. However, as shown in the paper, this approach can be circumvented in the case of SISO discrete–time systems by using dynamic output feedback controllers. At the present time, it is not known whether or not these results can be extended to the continuous–time case or MIMO systems.

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