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Author's reply: "A comment on $\mathscr{L}_\infty$ optimal control of SISO continuous-time systems"[☆]

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Abstract

In the above-mentioned comment, the author points out a technical problem with the paper (Wang, Z. Q., & Sznaier, M. (1997). *Automatica*, *33*(1), 85–90). As we show here, this technical problem can be easily solved. Moreover, it affects neither the main formulation nor the results, which remain valid. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In the paper (Yoon, 2000) the author points out a technical problem that arises in the context of using the space A_∞ to solve the $\mathscr{L}_\infty$ optimal control problem. Specifically, since transfer functions of the form $F(s) = M\mathrm{e}^{-st_0} \notin A_\infty$, then Lemma 1, as stated in Wang and Sznaier (1997) is incorrect. On this point we agree with the author and we thank him for pointing out this difficulty. However, as we show in the sequel, this problem can be easily solved by using the space A^e_∞ rather than A_∞ in the Lemma. Moreover, this affects neither the remainder of the paper nor its main result, which remains valid. Thus, we believe that the main claim made in Yoon (2000), namely that this technical problem invalidates the results of Wang and Sznaier (1997) and that the $\mathscr{L}_\infty$ control problem cannot be solved as proposed, is incorrect. Finally, we also strongly disagree with some of the remarks concerning the usefulness of the concept of $\mathscr{L}_\infty$-stability.

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2. On Lemma 1 and the Youla parametrization of $\mathscr{L}_\infty$ stabilizing controllers

In Yoon (2000), the author claims that since A_∞ is merely a commutative group and not a ring, it is hardly expected that the set of all $\mathscr{L}_\infty$-stable closed-loop systems can be obtained using the Youla parametrization. Moreover, it is claimed that in Wang and Sznaier (1997) the space A_∞ was endowed with the usual multiplication operation, and that it is doubtful that Lemma 1 can be proved as claimed. As we show next, these claims are incorrect. Note that in Wang and Sznaier (1997) it is never stated that A_∞ is a ring or a Banach algebra, only a linear space. Moreover, consider the following modified version of Lemma 1.

Lemma 1. *Assume that $U \in \mathscr{RH}_\infty$ has n-distinct zeros z_j in the open-right half plane and no zeros on the* jω*-axis. Let $M(s) = U(s)Q(s)$. Then $Q \in A^e_\infty$ if and only if $M \in \hat{T}^e$, where $\hat{T}^e = \{M \in A^e_\infty | M(z_j) = 0, j = 1, \ldots, n\}$.*

Proof. Necessity is immediate. To proof sufficiency, define

$$\hat{T}_\infty = \{M \in \mathscr{H}_\infty | M(z_j) = 0, j = 1, \ldots, n\}.$$

Note that any $M \in \hat{T}^e$ can be decomposed as $M = M_e M_\infty$, where $M_\infty \in \hat{T}_\infty$ and M_e has all its poles in the jω axis. From Theorem 2.2 in Callier and Desoer (1978) it follows that $Q_\infty \doteq M_\infty / U \in \mathscr{H}_\infty$. The proof

Nomenclature

$R(R_+)$	Set of real (positive real) numbers.
$\mathscr{L}_\infty(R_+)$	Banach space of Lebesgue measurable functions $x(t)$ on R_+, equipped with the $\|.\|_\infty$ norm.
$\mathscr{H}_\infty$	Banach space of complex functions with analytic continuation on the open right-half plane, and essentially bounded on the jω axis, equipped with the usual $\mathscr{H}_\infty$ norm $\|F\|_{\mathscr{H}_\infty} \doteq \text{esssup}_\omega \|F(j\omega)\|$.
A_∞	Space of Laplace transforms of elements in $\mathscr{L}_\infty$.
$\mathscr{A}^e_\infty$	Space of complex functions $F(s)$ analytic in $\text{Re}(s) > 0$, with a finite number of simple isolated singularities in $\text{Re}(s) = 0$ and such that $\|F(\infty)\| < \infty$.
$\|.\|_\infty$	$\mathscr{L}_\infty$ norm: $\|x\|_\infty \doteq \text{esssup}_{t\in R_+} \|x(t)\|$. By a slight abuse of notation, given $X(s) \in A_\infty$ we will use the notation $\|X(s)\|_\infty$ to denote $\|x(t)\|_\infty$, where $X(s) = \mathscr{L}[x(t)]$. Further, given $X(s) \in \mathscr{A}^e_\infty$ we define: $\|X\|_\infty = \begin{cases} \|x(t)\|_\infty & \text{if } X(s) \in A_\infty, \\ \infty & \text{otherwise.} \end{cases}$

follows now by noting that $Q \doteq M_e Q_\infty \in \mathscr{A}^e_\infty$ and $U*Q = M_e M_\infty = M$.

Next, we briefly show that indeed the set of achievable closed-loop systems $\Phi(s) \in \mathscr{A}^e_\infty$ can be parametrized as $\Phi(s) = H(s) - U(s)Q(s)$, $H, U \in \mathscr{H}_\infty$, $Q \in \mathscr{A}^e_\infty$. Proceeding as in Sanchez Pena and Sznaier (1998, Chapter 3), it can be easily shown that the set of all Full Information controllers such that the closed-loop system is in A^e_∞ is given by

$$C_{FI}(s) = [F \quad Q(s)], \quad Q \in A^e_\infty, \quad A + B_2 F \text{ stable}, \tag{1}$$

where A and B_2 denote the open-loop dynamics and control distribution matrix respectively. The proof for the general output feedback case follows from using an output injection to decompose the system into the cascade of an asymptotically stable system and a disturbance feedforward (equivalent to a FI) problem (see Sanchez Pena and Sznaier (1998) for details). □

3. On the solution of the $\mathscr{L}_\infty$ problem using duality

Next, we show that, contrary to the claim in Yoon (2000), the solution to the $\mathscr{L}_\infty$ control problem given in Wang and Sznaier (1997) is correct. Begin by noting that, without loss of generality, we can assume that $H(s)$ is strictly proper.[1] Consider now the following minimization problem:

$$\mu^* = \min_{m \in \hat{T}^e} \|h - m\|_\infty. \tag{2}$$

Since $H(s)$ is strictly proper, it follows that $\|h\|_\infty < \inf_{M \in A^e_\infty - A_\infty} \|h - m\|_\infty$. This, combined with the fact that $M(s) = 0 \in \mathscr{A}_\infty$ satisfies the interpolation constraints establishes that

$$\mu^* = \min_{m \in \hat{T}^e} \|h - m\|_\infty = \min_{m \in \hat{T}} \|h - m\|_\infty, \tag{3}$$

where $\hat{T} = \{M \in \mathscr{A}_\infty | M(z_j) = 0, j = 1, \ldots, n\}$. Thus the minor change in Lemma 1 ($A_\infty \to A^e_\infty$) does not entail any change in Theorem 2 in Wang and Sznaier (1997), which is correct, and indeed allows for finding the minimum $\mathscr{L}_\infty$ norm among the set of achievable $\mathscr{L}_\infty$-stable systems by recasting the problem into a minimum distance form and exploiting duality. Is it worth noticing that the extended Youla parameter (and the corresponding controller) may not be $\mathscr{L}_\infty$-stable. Indeed, this situation arises in Example 1 in Wang and Sznaier (1997). However, the closed-loop system is.

Finally, we disagree with the author's comment that the concept of $\mathscr{L}_\infty$-stability is not an "appropriate" concept from a system's theoretic standpoint, since the cascade of two $\mathscr{L}_\infty$ stable systems may not be $\mathscr{L}_\infty$-stable. Note that the concept of $\mathscr{L}_\infty$-stability is closely related to the concept of Lyapunov stability and that the series interconnection of Lyapunov stable systems is not necessarily Lyapunov stable. Of course, from a practical standpoint, one is interested in obtaining asymptotically (rather than $\mathscr{L}_\infty$)-stable systems. As we indicated in Wang and Sznaier (1997), this concept was used as an artifact to solve the problem, and asymptotic stability can be enforced by using suitable weights.

[1] If $H(s)$ is proper and $U(s)$ is strictly proper then the problem does not have a finite solution. If $U(s)$ is proper, we can always define $H_{\text{sp}} = H(s) - H(\infty) - U_{\text{sp}} H(\infty)/U(\infty)$, where U_{sp} denotes the strictly proper part of U, by appropriately shifting Q.

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