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An LMI Approach to Control-Oriented Identification and Model (In)Validation of LPV Systems

Mario Sznaier and María Cecilia Mazzaro

***Abstract*—This note proposes a control-oriented identification framework for a class of linear parameter varying systems that takes into account both the dependence of part of the model on time-varying parameters as well as the possible existence of a nonparametric component. The main results of the note show that the problems of obtaining and validating a model for these systems can be recast as linear matrix inequality feasibility problems. Moreover, as the information is completed, the algorithm is shown to converge in the ℓ_2-induced topology to the actual plant. Additional results include deterministic bounds on the identification error. These results are illustrated with a practical example arising in the context of active vision.**

***Index Terms*—Linear parameter varying (LPV) systems, control-oriented, robust identification and model (in)validation.**

I. Introduction

Motivated by the shortcomings of gain scheduling [11], during the past few years considerable attention has been devoted to the problem of synthesizing controllers for linear parameter varying (LPV) systems, where the state-space matrices of the plant depend on time-varying parameters that can be measured by the controller. Assuming that bounds on both the parameter values and their rate of change are known then affine matrix inequalities based conditions are available guaranteeing exponential stability of the system [2], [3].

Clearly, a key issue that needs to be addressed in order to apply these techniques to practical problems is the development of identification methods capable of extracting and validating the appropriate description from experimental data. Control-oriented identification of linear time-invariant (LTI) systems is by now relatively mature and efficient algorithms are available to obtain both models and worst-case bounds on the identification error [4]. On the other hand, identification tools for LPV systems are just starting to appear [5]–[8]. Moreover, at this point they bear more resemblance to classical identification methods (in the sense that they identify a set of parameters of a fixed structure) than to the control-oriented identification methods tailored to robust controls tools.

Motivated by our earlier results on control-oriented identification of LTI systems [9], in this note we propose a new robust identification framework for LPV systems that takes into account both the dependence of the dynamics on the time-varying parameters and the (possible) existence of a nonparametric part. The latter accounts for instance for dynamics not modeled by the parameter dependent portion of the model. The main results of this note show that the problems of establishing consistency of the experimental data with the *a priori* information and of obtaining and (in)validating a model of the system can be recast as linear matrix inequality (LMI) feasibility problems. Moreover, we show that as the information is completed, the algorithm converges, in the ℓ_2-induced topology, to the actual plant.

II. Notation

$\|\mathbf{x}\|_p$ denotes the p-norm of the real-valued column vector $\mathbf{x}$. $\mathbf{A}^T$ denotes the conjugate transpose of matrix $\mathbf{A}, \mathbf{A}(i,:)$ its i-th row and $\|\mathbf{A}\|_p$ its induced p-norm, $\mathbf{A}^\dagger$ its Moore-Penrose pseudoinverse and $\bar{\sigma}(\mathbf{A})$ its maximum singular value.

ℓp^m denotes the extended Banach space of vector-valued sequences x equipped with the p-norm. $\mathcal{P}_n$ denotes the nth step projection operator in ℓ_p^m, i.e., $\mathcal{P}_n(x) \doteq \{\mathbf{x}_0, \ldots, \mathbf{x}_{n-1}, \mathbf{0}, \mathbf{0}, \ldots\}$. $\mathcal{L}_\infty$ denotes the Lebesgue space of complex-valued matrix functions essentially bounded on the unit circle, equipped with the norm $\|G\|_\infty \doteq \operatorname{ess\,sup}_{|z|=1} \bar{\sigma}(G(z)); \mathcal{H}_\infty$ the subspace of functions in $\mathcal{L}_\infty$ with bounded analytic continuation inside the unit disk, equipped with the norm $\|G\|_\infty \doteq \operatorname{ess\,sup}_{|z|<1} \bar{\sigma}(G(z))$; and $\mathcal{H}_{\infty,\rho}$ the space of transfer matrices in $\mathcal{H}_\infty$ equipped with the norm $\|G\|_{\infty,\rho} \doteq \sup_{|z|<\rho} \bar{\sigma}(G(z))$. $\mathcal{BX}(\gamma)$ denotes the open γ-ball in a normed space $\mathcal{X}$, $\overline{\mathcal{BX}}(\gamma)$ its closure and $\mathcal{BX}(\overline{\mathcal{BX}})$ the open (closed) unit ball in $\mathcal{X}$.

$\mathcal{L}(\ell_2)$ denotes the space of discrete-time, LTV, single-input–single-output (SISO), causal and bounded operators in ℓ_2, equipped with the norm $\|H\|_{\ell_2\,\mathrm{ind}} \doteq \sup_{u\neq 0} \|Hu\|_2/\|u\|_2$. Any system of interest H will be represented by its convolution kernel $\{h_{i,j}\}$, by the finite lower triangular matrix $\mathbf{T}_H^n$ mapping sequences on the horizon $[0, n-1]$, where $\mathbf{T}_H^n(i,j) = h_{i,j}$ for $i \geq j$ and $\mathbf{T}_H^n(i,j) = 0$ otherwise, or by a minimal state-space realization $H \equiv \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$. In the particular case of LTI operators in $\mathcal{L}(\ell_2)$, we will also use the complex-valued transfer function $H(z) \doteq \sum_{k=0}^{\infty} h_k z^k$. This note considers SISO models, but all results can be extended to the multivariable case, following [10].

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Finally, given a metric space $\mathcal{X}$ equipped with the metric $m(x_1, x_2), d(\mathcal{A})$ denotes the diameter of set $\mathcal{A} \subseteq \mathcal{X}$, i.e., $d(\mathcal{A}) \doteq \sup_{x,a\in\mathcal{A}} m(x,a)$.

III. Control-Oriented Identification of LPV Systems

A. Problem Statement

Fig. 1(a) shows a diagram of the LPV system under consideration, denoted in short form as

$$G = \mathcal{F}_u(G_p, \Upsilon) + G_{\rm np} \tag{1}$$

where $\mathcal{F}_u$ stands for upper linear fractional transformation (LFT). Here, the signals u and y represent a known finite input sequence and its corresponding output corrupted by measurement noise η, collected in vectors $\mathbf{u}, \mathbf{y}$, and $\boldsymbol{\eta}$, respectively. For simplicity, it is assumed without loss of generality, that u is a finite unit impulse sequence applied at $k = 0$. The diagonal block $\Upsilon = \mathrm{diag}(v_1 I_{r_1}, \ldots, v_s I_{r_s})$ depends on a set of time-varying parameters, denoted by $v = \{\boldsymbol{v}_k\}_{k=0}^{\infty}$, that can be measured in real time.

Our goals are: 1) to identify a model G, consistent with both some *a priori* assumptions and the *a posteriori* experimental data, and 2) to obtain worst-case deterministic bounds on the identification error.

In the sequel we consider an *a priori* set of models $\mathcal{T}$ of the form (1), where the first term represents the parameter-varying portion of the dynamics, and the second one accounts for the (possible) existence of a nonparametric component. As usual in robust identification, we will assume that $G_{\rm np}$ belongs to the set $\mathcal{S}_{\rm np} \doteq \overline{\mathcal{BH}_{\infty,\rho}}(K)$ with $\rho > 1$ given, i.e., to the set of exponentially stable systems with a peak response to complex exponential inputs of K. Regarding the parameter-varying component, we will assume that it belongs to the set $\mathcal{S}_p$ of functions that admit an expansion of the form:

$$\mathcal{F}_u(G_p, \Upsilon) = \sum_{i=1}^{N_p} p_i \mathcal{F}_u(G_i, \Upsilon)$$

where p_i are unknown scalars and the known transfer matrices $G_i(z)$ are such that the impulse responses of the N_p interconnections $\mathcal{F}_u(G_i, \Upsilon)$ are linearly independent for all admissible parameter trajectories. We will further assume that the system G is exponentially stable.

Remark 1: As we show in the sequel, the linear independence assumption is required to establish global convergence of the method. Intuitively, it guarantees that the impulse experiment is "rich" enough to identify the system. If it fails, a different input should be used. A deeper discussion of this condition and possible relaxations will be presented in Section III-B.2.

Finally, we will consider *a priori* noise of the form

$$\mathcal{N} \doteq \left\{ \boldsymbol{\eta} \in \Re^N : \mathbf{L}(\boldsymbol{\eta}) = \mathbf{L}_0 + \sum_{k=1}^{N} \mathbf{L}_k \eta_{k-1} \geq 0 \right\} \tag{2}$$

where $\mathbf{L}_i$ are given real-valued symmetric matrices. This noise set is a generalization of the set $\{\boldsymbol{\eta} \in \Re^N : |\eta_k| \leq \epsilon\}$ usually considered in the literature, that allows for taking into consideration correlated noise. As we illustrate in Section V, models of the form (1)–(2) arise, for instance, in the context of active vision applications.

Definition 1: Given the experiments $(\mathbf{y}, \boldsymbol{\Upsilon})$, the consistency set $\mathcal{T}(\mathbf{y}, \boldsymbol{\Upsilon})$ is defined as the set of all possible models compatible with the *a priori* assumptions $(\mathcal{T}, \mathcal{N})$, that could have generated the *a posteriori* experimental data $\mathbf{y}$, i.e.,

$$\mathcal{T}(\mathbf{y}, \boldsymbol{\Upsilon}) \doteq \{G \in \mathcal{T} : \mathbf{y} - \mathbf{T}_G^N \mathbf{u} \in \mathcal{N}\}.$$

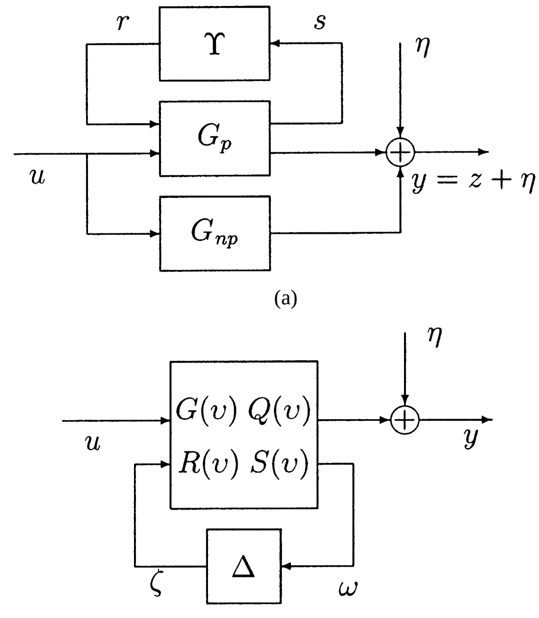


Fig. 1. (a) LPV control-oriented identification and (b) model (in)validation setups.

Using this definition the LPV identification problem can be precisely stated as follows.

Problem 1: Given the experiments $(\mathbf{y}, \boldsymbol{\Upsilon})$ and the *a priori* sets $(\mathcal{T}, \mathcal{N})$

i) determine whether the *a priori* and *a posteriori* information are consistent, i.e., whether $\mathcal{T}(\mathbf{y}, \boldsymbol{\Upsilon}) \neq \emptyset$;

ii) if $\mathcal{T}(\mathbf{y}, \boldsymbol{\Upsilon}) \neq \emptyset$, find a nominal model $G \in \mathcal{T}(\mathbf{y}, \boldsymbol{\Upsilon})$ and a worst-case bound on the identification error.

In the sequel, we will show that these problems can be recast as an LMI feasibility problem.

B. Main Results

1) Consistency and Identification: In this section we will solve the consistency problem by reducing it to a Carathéodory–Fejér interpolation problem and showing that the latter is equivalent to an LMI feasibility problem.

Theorem 1: The *a priori* and *a posteriori* information are consistent if and only if there exist two vectors $\mathbf{p} = [p_1 \ \ldots \ p_{N_p}]^T$ and $\mathbf{h} = [h_0 \ \cdots \ h_{N-1}]^T$ such that:

$$\mathbf{M}_R(\mathbf{h}) \geq 0 \quad (\mathbf{y} - \mathbf{P}_N \mathbf{p} - \mathbf{h}) \in \mathcal{N} \tag{3}$$

where

$$\mathbf{M}_R(\mathbf{h}) \doteq \begin{bmatrix} \mathbf{R}^{-2} & \frac{1}{K}\mathbf{F}^T \\ \frac{1}{K}\mathbf{F} & \mathbf{R} \end{bmatrix} \quad \mathbf{P}_N \doteq \begin{bmatrix} g_0^1 & \cdots & g_0^{N_p} \\ \vdots & \ddots & \vdots \\ g_{N-1}^1 & \cdots & g_{N-1}^{N_p} \end{bmatrix}$$

$$\mathbf{F} \doteq \begin{bmatrix} h_0 & \cdots & h_{N-1} \\ \vdots & \cdots & \vdots \\ 0 & \cdots & h_0, \end{bmatrix}$$

$\mathbf{R} \doteq \mathrm{diag}(1, \rho, \ldots, \rho^{N-1})$ and g_m^i denotes the mth element of the impulse response of $\mathcal{F}_u(G_i, \Upsilon)$.

Proof: Given the parameter trajectory $\mathbf{\Upsilon}$, the experimental data $\mathbf{y}$ is consistent with the *a priori* information if and only if there exist vectors $\mathbf{p}, \mathbf{h}, \boldsymbol{\eta} = [\eta_0 \ \ldots \ \eta_{N-1}]^T$ and a function $H(z) \in \overline{\mathcal{BH}_{\infty,\rho}}(K)$ with $\rho > 1$ such that:

$$y_k = \eta_k + h_k + \sum_i^{N_p} p_i g_k^i, \qquad \boldsymbol{\eta} \in \mathcal{N}$$
$$H(z) = h_o + h_1 z + \cdots + h_{N-1} z^{N-1} + \cdots. \tag{4}$$

Note that $H(z) \in \overline{\mathcal{BH}_{\infty,\rho}}(K)$ if and only if $\hat{H}(z) = H(\rho z)/K \in \overline{\mathcal{BH}_{\infty}}$, with coefficients $\hat{h}_k = \rho^k h_k/K$. By applying Carathéodory–Fejér interpolation theory, the existence of $H(z)$ is equivalent, after some algebra, to $\mathbf{R}^{-2} - (1/K^2)\mathbf{F}^T\mathbf{R}^{-2}\mathbf{F} \geq 0$ [11, Ch. 18]. The first inequality in (3) follows now from Schur complements; the second one is simply a restatement of (4). ■

Once consistency is established a nominal model can be obtained proceeding as follows.

i) Find a pair of data vectors $\mathbf{p}, \mathbf{h}$ satisfying the LMIs (3).
ii) All solutions G_{np} (nonparametric component) can be parametrized as a lower LFT of a free parameter $Q(z)$ (see [11, Ch. 18] for details). In particular the choice $Q(z)$ yields the central solution $G_{\mathrm{np}} = KT_1(z/\rho)T_2^{-1}(z/\rho)$, where the transfer function $T(z)$ has the following state-space realization:

$$T(z) = [T_1(z) \quad T_2(z)]^T \equiv \{\mathbf{A}_T, \mathbf{B}_T, \mathbf{C}_T, \mathbf{D}_T\}$$
$$\mathbf{A}_T = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{(N-1)\times(N-1)} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{B}_T = \mathbf{M}_P^{-1}\left(\mathbf{A}_T^T - \mathbf{I}\right)^{-1}[\,\mathbf{C}_-^T\,]$$
$$\mathbf{C}_T = \begin{bmatrix} \mathbf{C}_+ \\ \mathbf{C}_- \end{bmatrix}(\mathbf{A}_T - \mathbf{I})$$
$$\mathbf{D}_T = \mathbf{I} + \begin{bmatrix} \mathbf{C}_+ \\ \mathbf{C}_- \end{bmatrix}\mathbf{M}_P^{-1}\left(\mathbf{A}_T^T - \mathbf{I}\right)^{-1}[\,\mathbf{C}_-^T\,]$$
$$\mathbf{C}_- = [1 \quad 0 \quad \cdots \quad 0],\ \mathbf{C}_+ = [h_0 \quad \cdots \quad h_{N-1}].$$

The complete model is given by $G_{\mathcal{A}} = G_{\mathrm{np}} + \sum_{i=1}^{N_p} p_i \mathcal{F}_u(G_i, \Upsilon)$.

2) Analysis of the Identification Error and Convergence: Next, we show that the proposed algorithm is convergent and we derive some worst-case bounds on the identification error. Begin by noting that the proposed algorithm is interpolatory (in the sense that it always generates a model inside the consistency set $\mathcal{T}(\mathbf{y}, \mathbf{\Upsilon})$) and recall that, for *any* interpolatory algorithm $\mathcal{A}$, the worst-case identification error is bounded by (see, for instance, [4, Ch. 10])

$$e(\mathcal{A}) \leq \mathcal{D}(\mathcal{I})$$

where $\mathcal{D}(\mathcal{I})$ denotes the *diameter* of information.

Note that in contrast to the case of LTI systems, here the experiment operator $y = \mathrm{E}(G, \eta, u, v)$ that maps the model, inputs and noise to the experimental outcome is not linear (since, in general, the plant depends nonlinearly on the time-varying parameters v) and, thus, $\mathcal{D}(\mathcal{I})$ may not be easily computable. To circumvent this difficulty, we introduce the concepts of *parameter-dependent* diameter of information and identification error. These concepts will be used to establish convergence of the algorithm for all parameter trajectories.

Definition 2: Given a parameter trajectory $\hat{\mathbf{\Upsilon}} = [\hat{\boldsymbol{v}}_0 \ \ldots \ \hat{\boldsymbol{v}}_{N-1}]$, define the set $\mathcal{Y}(\hat{\mathbf{\Upsilon}})$ as the set of all experimental data consistent with the *a priori* information, for all possible parameter trajectories compatible with the first N measurements $\hat{\mathbf{\Upsilon}}$. The parameter-dependent diameter of information is given by

$$\mathcal{D}(\mathcal{I}, \hat{\mathbf{\Upsilon}}) \doteq \sup_{\mathbf{y}\in\mathcal{Y}, \Upsilon\in\mathbf{\Gamma}} d(\mathcal{T}(\mathbf{y}, \mathbf{\Upsilon}))$$

where $\mathbf{\Gamma} \doteq \{v: \boldsymbol{v}_k = \hat{\boldsymbol{v}}_k,\ k = 0, 1, \ldots, N-1\}$, i.e., the "size" of the largest set of indistinguishable models compatible with the *a priori* information and the first N measurements of the parameter trajectory.

Similarly, we can define a *parameter-dependent* identification error by considering the worst-case error over all trajectories compatible with the present parameter trajectory.

Definition 3: Given the experimental outcome $(\mathbf{y}, \hat{\mathbf{\Upsilon}})$, let $G_{\mathcal{A}} = \mathcal{A}(\mathbf{y}, \hat{\mathbf{\Upsilon}})$ denote the nominal model obtained using the algorithm proposed in Section III.B.1. The parameter-dependent identification error is defined as:

$$e(\mathcal{A}, \hat{\mathbf{\Upsilon}}) \doteq \sup_{G\in\mathcal{T}(\mathbf{y},\hat{\mathbf{\Upsilon}})} \|G - G_{\mathcal{A}}\|_{\ell_2\,\mathrm{ind}}.$$

Since for a given $\hat{\mathbf{\Upsilon}}$ the sets $\mathcal{N}$ and $\mathcal{T}(\hat{\mathbf{\Upsilon}}) = \{G: G = G_{\mathrm{np}} + \sum_{i=1}^{N_p} p_i \mathcal{F}_u(G_i, \hat{\Upsilon})\}$ are convex and symmetric with respect to the points $G = 0, \boldsymbol{\eta} = \mathbf{0}$, it follows (see [4, Lemma 10.2]) that, as in the nonparametric case, the worst-case experiment is the one that yields a zero outcome, i.e.,

$$e(\mathcal{A}, \hat{\mathbf{\Upsilon}}) \leq \mathcal{D}(\mathcal{I}, \hat{\mathbf{\Upsilon}}) = 2 \sup_{G\in\mathcal{T}(\mathbf{0},\hat{\mathbf{\Upsilon}})} \|G\|_{\ell_2\,\mathrm{ind}}. \tag{5}$$

In order to show convergence of the proposed algorithm, we need the following additional assumption.

Assumption 1: There exists some N^* large enough, but independent of Υ, such that for all parameter trajectories and all elements $G_{\mathrm{np}} \in \mathcal{S}_{\mathrm{np}}/\{0\}$ the matrix

$$\mathbf{P}_{\mathrm{aug}}(\Upsilon) \doteq [\mathbf{P}_{N^*} \quad \mathbf{g}_{N^*}^{\mathrm{np}}] \quad \mathbf{g}_{N^*}^{\mathrm{np}} \doteq [g_0^{\mathrm{np}} \quad \cdots \quad g_{N^*}^{\mathrm{np}}]^T$$

has full-column rank, with $\mathbf{P}_{N^*}$ defined as in Theorem 1, $\{g^{\mathrm{np}}\} = \mathrm{imp}(G_{\mathrm{np}})$ and where $\mathrm{imp}(\cdot)$ denotes impulse response.

This assumption essentially rules out the existence of arbitrarily large time delays in the non parametric portion of the models. In Section III-C, we will analyze some of its implications and discuss possible relaxations.

For simplicity in the sequel we will assume that the *a priori* noise set is of the form $\mathcal{N} = \{\boldsymbol{\eta} \in \Re^N: |\eta_k| \leq \epsilon\}$ but the error bounds and convergence proof easily generalize to sets of the form (2).

Theorem 2: If assumption 1 holds, then the algorithm is convergent, i.e.,

$$\lim_{N\to\infty, \mathcal{N}\to\{\mathbf{0}\}} \|G_o - G_{\mathcal{A}}\|_{\ell_2\,\mathrm{ind}} = 0$$

where $G_o \doteq G_{o_{\mathrm{np}}} + \sum_{i=1}^{N_p} p_{o_i} \mathcal{F}_u(G_i, \Upsilon)$ denotes the actual system.

Proof: The proof is divided in two parts: 1) establishing that $e(\mathcal{A}, \hat{\mathbf{\Upsilon}}) \to 0$, and 2) using this last result to show that $\|G_o - G_{\mathcal{A}}\|_{\ell_2\,\mathrm{ind}} \to 0$.

To prove 1), consider sequences $N_i \uparrow \infty, \epsilon_i \downarrow 0$, and for a given pair (N, ϵ) denote by $\mathcal{T}(\mathbf{y}_0, N, \epsilon, \hat{\mathbf{\Upsilon}})$ the set of plants consistent with the *a priori* information, the observed parameter trajectory $\hat{\mathbf{\Upsilon}}$ and the null outcome $\mathbf{y}_0$. Clearly, if $\epsilon_j < \epsilon_i$ and $N_j \geq N_i$, then $\mathcal{T}(\mathbf{y}_0, N_j, \epsilon_j, \hat{\mathbf{\Upsilon}}_j) \subset \mathcal{T}(\mathbf{y}_0, N_i, \epsilon_i, \hat{\mathbf{\Upsilon}}_i)$ and, thus, [12, p. 18] the sequence of sets $\mathcal{T}(\cdot)$ has a limit $\tilde{\mathcal{T}}(\hat{\mathbf{\Upsilon}}) = \cap_k \mathcal{T}(\mathbf{y}_0, N_k, \epsilon_k, \hat{\mathbf{\Upsilon}}_k)$. If $\tilde{\mathcal{T}}(\hat{\mathbf{\Upsilon}}) \neq \{0\}$, then there exists some $0 \neq \tilde{G} = \sum_{i=1}^{N_p} \tilde{p}_i \mathcal{F}_u(G_i, \Upsilon) + \tilde{G}_{\mathrm{np}} \in \mathcal{T}(\mathbf{y}_0, N_j, \epsilon_j, \hat{\mathbf{\Upsilon}}_j), \forall j$. Assume first that $\tilde{G}_{\mathrm{np}} \neq 0$, and define $\{\tilde{g}_{\mathrm{np}}\} = \mathrm{imp}(\tilde{G}_{\mathrm{np}}), \hat{K} = \|\mathcal{P}_{N*}(\tilde{g}_{\mathrm{np}})\|_2, \{\hat{g}^{\mathrm{np}}\} = \mathrm{imp}(\tilde{G}_{\mathrm{np}}/\hat{K})$. From Assumption 1, it follows that the matrix

$$\hat{\mathbf{P}}_{\mathrm{aug}}(\Upsilon) \doteq [\mathbf{P}_{N^*} \quad \hat{\mathbf{g}}_{N^*}^{\mathrm{np}}] \quad \hat{\mathbf{g}}_{N^*}^{\mathrm{np}} \doteq [\hat{g}_0^{\mathrm{np}} \quad \cdots \quad \hat{g}_{N^*}^{\mathrm{np}}]^T$$

has full-column rank. Since $\tilde{G} \in \mathcal{T}(\mathbf{y}_0, N^*, \epsilon, \hat{\mathbf{\Upsilon}}_k)$, it follows that $\|\,[\,\tilde{\mathbf{p}}^T \quad \hat{K}\,]^T\,\|_\infty \leq \|\hat{\mathbf{P}}_{\mathrm{aug}}^\dagger\|_1 \epsilon$. Thus, $\|\tilde{G}\|_{\ell_2\,\mathrm{ind}} \to 0$ as $\epsilon \to 0$, which contradicts the fact that $\tilde{G} \neq 0$. If $\tilde{G}_{\mathrm{np}} = 0$, a similar argument holds considering the submatrix of $\hat{\mathbf{P}}_{\text{aug}}$ formed by its first N_p columns. The desired result follows now directly from (5).

To prove convergence, note that, from the definition of $e(\mathcal{A}, \hat{\mathbf{\Upsilon}})$ we have that $\|G_o - G_{\mathcal{A}}\|_{\ell_2 \,\text{ind}} \leq e(\mathcal{A}, \hat{\mathbf{\Upsilon}}) \to 0$. This equation, combined with the linear independence of the impulse responses of the N_p interconnections $\mathcal{F}_u(G_i, \Upsilon)$ and the separation condition (Assumption 1) implies that $\|G_{\text{np}} - G_{o_{\text{np}}}\|_{\ell_2 \,\text{ind}} \to 0$ and $p_i \to p_{o_i}$. ∎

As we show next, a bound on $\mathcal{D}(\mathcal{I}, \hat{\mathbf{\Upsilon}})$ can be obtained by solving an LMI optimization problem.

Theorem 3: The parameter-dependent diameter of information can be bounded above by

$$\mathcal{D}(\mathcal{I}, \hat{\mathbf{\Upsilon}}) \leq 2\left[a\|\mathbf{p}\|_\infty + \sum_{i=0}^{N-1} \nu_i + K\rho^{-(N-1)}/(\rho - 1)\right] \quad (6)$$

$$a \doteq \sum_{i=1}^{N_p} \sup_{\upsilon \in \mathbf{\Gamma}} \|\mathcal{F}_u(G_i, \Upsilon)\|_{\ell_2 \,\text{ind}}$$

$$\nu_i \doteq \min\{K\rho^{-i}, \|\mathbf{P}_N(i, :)\|_1 \|\mathbf{p}\|_\infty + \epsilon\}.$$

Proof: Since $G \in \mathcal{T}(\mathbf{0}, \hat{\mathbf{\Upsilon}})$, it follows that

$$\mathbf{P}_N \mathbf{p} + \mathbf{h} \in \mathcal{N} \quad (7)$$

where $\mathbf{P}_N, \mathbf{p}$, and $\mathbf{h}$ are defined in Theorem 1. By assumption, the impulse responses of the N_p interconnections $\mathcal{F}_u(G_i, \Upsilon)$ are linearly independent for any given trajectory of the time-varying parameters. Thus, for N large enough $\mathbf{P}_N$ has full-column rank, i.e., it has a left inverse $\mathbf{P}_N^\dagger$. Hence, $\mathbf{p} = \mathbf{P}_N^\dagger(\boldsymbol{\eta} - \mathbf{h})$ for some $\boldsymbol{\eta} \in \mathcal{N}$. It follows that $\|\mathbf{p}\|_\infty \leq \|\mathbf{P}_N^\dagger\|_1 \|(\boldsymbol{\eta} - \mathbf{h})\|_\infty \leq \|\mathbf{P}_N^\dagger\|_1(\epsilon + K)$ where the last inequality follows from the fact that $G_{\text{np}} \in \overline{\mathcal{BH}_{\infty,\rho}}(K)$. Moreover, this fact, combined with (7) implies that $|h_i| \leq \nu_i$. Thus, for every $G \in \mathcal{T}(\mathbf{0}, \hat{\mathbf{\Upsilon}})$ we have that

$$\begin{aligned}\|G\|_{\ell_2 \,\text{ind}} &\leq \sum_{i=1}^{N_p} |p_i| (\sup_{\upsilon \in \mathbf{\Gamma}} \|\mathcal{F}_u(G_i, \Upsilon)\|_{\ell_2 \,\text{ind}}) + \|G_{\text{np}}\|_{\ell_2 \,\text{ind}} \\ &\leq a\|\mathbf{p}\|_\infty + \sum_{i=0}^{N-1} \nu_i + K\rho^{-(N-1)}/\rho - 1.\end{aligned}$$

∎

Consider a state-space realization for the LFT $\mathcal{F}_u(G_i, \Upsilon)$

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{A}(\boldsymbol{\upsilon}_k)\mathbf{x}_k + \mathbf{B}(\boldsymbol{\upsilon}_k)\mathbf{u}_k \\ \mathbf{z}_k &= \mathbf{C}(\boldsymbol{\upsilon}_k)\mathbf{x}_k + \mathbf{D}(\boldsymbol{\upsilon}_k)\mathbf{u}_k.\end{aligned} \quad (8)$$

As shown next, an upper bound on $\sup_{\upsilon \in \mathbf{\Gamma}} \|\mathcal{F}_u(G_i, \Upsilon)\|_{\ell_2 \,\text{ind}}$, and therefore on constant a defined in (6), can be computed by solving a (functional) affine matrix inequality optimization problem.

Lemma 1: Assume that the set of admissible parameter trajectories is of the form $\mathcal{F}_\Theta = \{\upsilon: \boldsymbol{\upsilon}_{k+1} \in \Theta[\boldsymbol{\upsilon}_k], k = 0, 1, \ldots\}$, where $\Theta: \mathcal{P} \rightsquigarrow \mathcal{P}$ is a given set valued map. If the following parameter-dependent affine matrix inequality in the variable $\mathbf{X}(\upsilon) > 0$:

$$\begin{bmatrix} \mathbf{A}^T(\upsilon)\mathbf{X}(\theta)\mathbf{A}(\upsilon) - \mathbf{X}(\upsilon) & \mathbf{A}^T(\upsilon)\mathbf{X}(\theta)\mathbf{B}(\upsilon) & \mathbf{C}^T(\upsilon) \\ \mathbf{B}^T(\upsilon)\mathbf{X}(\theta)\mathbf{A}(\upsilon) & \mathbf{B}^T(\upsilon)\mathbf{X}(\theta)\mathbf{B}(\upsilon) - \gamma^2\mathbf{I} & \mathbf{D}^T(\upsilon) \\ \mathbf{C}(\upsilon) & \mathbf{D}(\upsilon) & -\mathbf{I} \end{bmatrix} < 0 \quad (9)$$

holds for all $\upsilon \in \mathcal{P}$ and $\theta \in \Theta(\upsilon)$, then $\|\mathcal{F}_u(G_i, \Upsilon)\|_{\ell_2 \,\text{ind}} \leq \gamma$.

Proof: Pre- and postmultiplying (9) by $[\mathbf{x}_k^T \ \mathbf{u}_k^T \ \mathbf{z}_k^T]$ and its transpose, we have for every admissible parameter trajectory

$$0 > \mathbf{x}_{k+1}^T \mathbf{X}(\boldsymbol{\upsilon}_{k+1})\mathbf{x}_{k+1} - \mathbf{x}_k^T \mathbf{X}(\boldsymbol{\upsilon}_k)\mathbf{x}_k + \mathbf{z}_k^T \mathbf{z}_k - \gamma^2 \mathbf{u}_k^T \mathbf{u}_k.$$

Summing this last equation from $k = 0$ to ∞ and using the fact that $\lim_{k\to\infty} \mathbf{x}_k = \mathbf{0}$ due to exponential stability of the system yields:

$$\|z\|_2^2 = \sum_{k=0}^{\infty} \mathbf{z}_k^T \mathbf{z}_k < \gamma^2 \sum_{k=0}^{\infty} \mathbf{u}_k^T \mathbf{u}_k = \gamma^2 \|u\|_2^2.$$

∎

Remark 2: The set valued map Θ is a generalization of the usual rate bounds $\underline{\nu} \leq \dot{\upsilon}_i \leq \overline{\nu}$, that allows for considering for instance discrete parameter values and parameter variations with memory. In the case of arbitrarily fast varying parameters, $\Theta(\upsilon) = \mathcal{P}$.

Since $\mathbf{A}(\upsilon), \mathbf{B}(\upsilon), \mathbf{C}(\upsilon)$ and $\mathbf{D}(\upsilon)$ are affine in υ it follows that in the simpler case of polytopic $\mathcal{P}$ and arbitrarily fast varying parameters, checking that (9) holds can be accomplished by just checking the vertices of $\mathcal{P}$. As is the case with standard LPV analysis, more complex dependencies may require griding the parameter set.

Remark 3: In many cases, the bound given in (6) can be quite conservative. A tighter bound can be obtained, at the expense of increased computational complexity, by solving the following convex optimization problem:

$$\begin{aligned}&\max_{\mathbf{p},\mathbf{h}} \left\{ \sup_{\upsilon \in \mathbf{\Gamma}} \left\| \sum_{i=1}^{N_p} p_i \mathcal{F}_u(G_i, \Upsilon) \right\|_{\ell_2 \text{ind}} + \|\mathbf{h}\|_1 + K\frac{\rho^{-(N-1)}}{\rho - 1} \right\} \\ &\text{subject to: } \mathbf{P}_N \mathbf{p} + \mathbf{h} \in \mathcal{N} \\ &\qquad |h_i| \leq K\rho^{-i}.\end{aligned} \quad (10)$$

Note that, for each value of $\mathbf{p}$, the objective function can be computed by first finding a state space realization of $\sum_i p_i \mathcal{F}_u(G_i, \Upsilon)$ and then applying Lemma 1.

C. Relaxing Some of the Assumptions

In this section, we briefly address the issue of relaxing some of the assumptions made before.

Linear independence on the impulse responses of the N_p interconnections $\mathcal{F}_u(G_i, \Upsilon)$. This assumption can be relaxed if a bound on $\|\mathbf{p}\|_\infty$ is available as part of the *a priori* information. In that case this bound can be directly used in Theorem 3 to establish the bound on $\mathcal{D}(\mathcal{I}, \hat{\mathbf{\Upsilon}})$.

Separation condition (Assumption 1). This condition is required in order to guarantee convergence of the identified model. It can be relaxed to hold only on certain parameter trajectories, provided that the identification is performed over one of these. Intuitively, this condition guarantees a unique parametric/nonparametric decomposition for each element $G_{\text{np}} + \sum_{i=1}^{N_p} p_i \mathcal{F}_u(G_i, \Upsilon)$ of the *a priori* set. If it fails for a given parameter trajectory $\hat{\mathbf{\Upsilon}}$, then for this trajectory there exist multiple parameter choices. Therefore, the parametric and nonparametric components will not converge separately, although the full model might converge to the real plant. Since by assumption the model to be identified consists of a nonparametric and a parameter-dependent portion, it is reasonably to assume that this condition will hold for most of the parameter trajectories. If this is not the case, then the nonparametric part can be absorbed into the parametric one.

IV. (In)Validation of LPV Models

Next, we turn our attention to the related problem of model (in)validation. Note that the error bounds provided in the previous section, while useful to establish the convergence properties of the algorithm, tend to be too conservative for control synthesis. Moreover, from a practical standpoint, before using the identified model and associated uncertainty description to synthesize controllers, they should be validated using *new* data, that has not been used in the identification process (to avoid introducing biases).

Consider the lower fractional interconnection, shown on Fig. 1(b), between a discrete-time stable LPV model $P(v)$ and an unstructured LTV uncertainty Δ in the set $\mathbf{\Delta} \doteq \overline{\mathcal{BL}(\ell_2)}(\delta)$. The block $P(v)$ consists of a nominal model of the physical system $G(v)$—obtained for example using the method proposed in Section III—and a description, given by the blocks $Q(v)$, $R(v)$, and $S(v)$, of how uncertainty enters the model. We will further assume that $\|S(v)\|_{\ell_2\,\text{ind}} < \delta^{-1}$ holds for all parameter trajectories v so that the interconnection $\mathcal{F}_l(P(v), \Delta)$ is ℓ_2 stable for all v. Finally, the signals u and y represent an arbitrary but known test input and its corresponding output respectively, corrupted by measurement noise $\eta \in \mathcal{N}$, where $\mathcal{N}$ is of the form (2). As before, their values over a finite horizon are collected in vectors $\mathbf{u}, \mathbf{y}$, and $\boldsymbol{\eta}$.

The goal is to determine whether or not the measured values of the input u, the output y and the time-varying parameters v are consistent with the assumed model $P(v)$ and the given set descriptions for the noise $\boldsymbol{\eta} \in \mathcal{N}$ and uncertainty $\Delta \in \mathbf{\Delta}$, that is, the following.

Problem 2: Given the time-domain experiments

$$\begin{aligned}\mathbf{u} &\doteq [\,u_0 \quad u_1 \quad \ldots \quad u_{n-1}\,]^T\\ \mathbf{\Upsilon} &\doteq [\,\boldsymbol{v}_0 \quad \boldsymbol{v}_1 \quad \ldots \quad \boldsymbol{v}_{n-1}\,]\\ \mathbf{y} &\doteq [\,y_0 \quad y_1 \quad \ldots \quad y_{n-1}\,]^T\end{aligned}$$

the model $P(v)$ and the *a priori* sets $\mathcal{N}, \mathbf{\Delta}$ determine whether or not the *a priori* and *a posteriori* information are consistent, i.e., whether the set

$$\mathcal{M}(\mathbf{y}, \mathbf{\Upsilon}, P) = \left\{(\Delta, \boldsymbol{\eta})\colon \Delta \in \mathbf{\Delta}, \qquad \boldsymbol{\eta} \in \mathcal{N}, \mathbf{y} = \mathbf{T}^n_{\mathcal{F}_l(P(v),\Delta)}\mathbf{u} + \boldsymbol{\eta}\right\}$$

is nonempty.

Next, we recast the (in)validation problem subject to LTV uncertainty into LMI feasibility form.

Theorem 4: Given time-domain measurements of the input $\mathbf{u}$, the output $\mathbf{y}$ and the time-varying parameters $\mathbf{\Upsilon}$, the LPV model $P(v)$ is not invalidated by this experimental information if and only if there exist two vectors $\boldsymbol{\zeta} = [\zeta_0 \ \ldots \ \zeta_{n-1}]^T$ and $\boldsymbol{\eta} = [\eta_0 \ \ldots \ \eta_{n-1}]^T$, such that the following set of $(n+1)$ LMIs hold:

$$\mathbf{M}^k(\boldsymbol{\zeta}) > 0 \ k = 1, \ldots, n, \qquad \mathbf{L}(\boldsymbol{\eta}) > 0 \tag{11}$$

where

$$\begin{aligned}\mathbf{M}^k(\boldsymbol{\zeta}) &\doteq \begin{bmatrix} \mathbf{X}^k(\boldsymbol{\zeta}) & (\boldsymbol{\zeta}^k)^T \\ \boldsymbol{\zeta}^k & \mathbf{Y}^k(\boldsymbol{\zeta}) \end{bmatrix}\\ \mathbf{Y}^k(\boldsymbol{\zeta}) &\doteq \left(\frac{1}{\delta^2}\mathbf{I} - \left(\mathbf{T}^k_S\right)^T \mathbf{T}^k_S\right)^{-1}\\ \mathbf{X}^k(\boldsymbol{\zeta}) &\doteq \left(\mathbf{T}^k_R\mathbf{u}^k\right)^T \mathbf{T}^k_R\mathbf{u}^k + \left(\mathbf{T}^k_R\mathbf{u}^k\right)^T \mathbf{T}^k_S\boldsymbol{\zeta}^k\\ &\quad + \left(\mathbf{T}^k_S\boldsymbol{\zeta}^k\right)^T \mathbf{T}^k_R\mathbf{u}^k\\ \boldsymbol{\eta} &\doteq \mathbf{y} - \mathbf{T}^n_G\mathbf{u} - \mathbf{T}^n_Q\boldsymbol{\zeta}\end{aligned} \tag{12}$$

and $\mathbf{L}(\boldsymbol{\eta})$ is defined as in (2). Here, vectors $\mathbf{u}^k, \boldsymbol{\zeta}^k$, and $\boldsymbol{\omega}^k$ contain the first k elements of sequences u, ζ, and ω, respectively.

Proof: The LPV model $P(v)$ is not invalidated by the experimental information $\{\mathbf{u}, \mathbf{y}, \mathbf{\Upsilon}\}$ if and only if there exist a $\Delta \in \mathbf{\Delta}$ and a $\boldsymbol{\eta} \in \mathcal{N}$ such that

$$\begin{aligned}\mathbf{y} &= \mathbf{T}^n_G\mathbf{u} + \mathbf{T}^n_Q\boldsymbol{\zeta} + \boldsymbol{\eta}\\ \omega &= \mathbf{T}^n_R\mathbf{u} + \mathbf{T}^n_S\boldsymbol{\zeta} \quad \boldsymbol{\zeta} = \mathbf{T}^n_\Delta\boldsymbol{\omega}.\end{aligned} \tag{13}$$

Using [13, Th. 4.5], the existence of an uncertainty block in $\mathbf{\Delta}$ is equivalent to

$$(\boldsymbol{\zeta}^k)^T\boldsymbol{\zeta}^k \le \delta^2(\boldsymbol{\omega}^k)^T\boldsymbol{\omega}^k, 1c \quad k = 1, \ldots, n. \tag{14}$$

Now, replacing the expression of $\boldsymbol{\omega}^k$ from (13) into the right-hand side of each of the inequalities (14), and reordering terms yields

$$\begin{aligned}(\boldsymbol{\zeta}^k)^T\left(\frac{1}{\delta^2}\mathbf{I} - \left(\mathbf{T}^k_S\right)^T\mathbf{T}^k_S\right)\boldsymbol{\zeta}^k &\le \left(\mathbf{T}^k_R\mathbf{u}^k\right)^T\mathbf{T}^k_R\mathbf{u}^k +\\ &+ \left(\mathbf{T}^k_R\mathbf{u}^k\right)^T\mathbf{T}^k_S\boldsymbol{\zeta}^k + (\boldsymbol{\zeta}^k)^T\left(\mathbf{T}^k_S\right)^T\mathbf{T}^k_R\mathbf{u}^k\end{aligned} \tag{15}$$

for $k = 1, \ldots, n$. Using Schur complements and the fact that $\|S(v)\|_{\ell_2\,\text{ind}} < \delta^{-1}$ gives the first n LMI's of the set (11), $\mathbf{M}^k(\boldsymbol{\zeta}) > 0, k = 1, \ldots, n$. The last LMI of (11), $\mathbf{L}(\boldsymbol{\eta}) > 0$, is simply obtained by replacing the expression of the noise vector $\boldsymbol{\eta}$ from (12) in the definition of $\mathcal{N}$ given in (2). ■

V. A Practical Example

In this section, we illustrate the proposed framework with a practical example arising in the context of active vision. Consider the problem of smooth tracking of a noncooperative target, illustrated in the block diagram shown in Fig. 2(a). Here, the goal is to internally stabilize the plant and to track target motions, y_{target}, using as measurements images possibly corrupted by noise, while zooming in and out of features of interest.

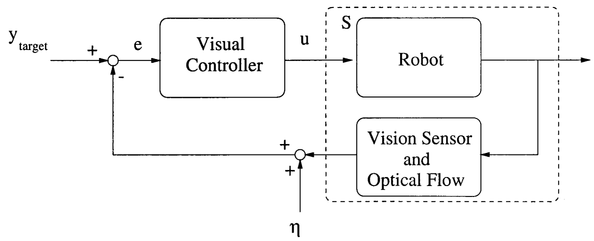
Designing a controller for this application requires, as a first step, finding a model for the block labeled S in Fig. 2(a) that maps the command input to the head, in encoder units, to the position of the target (in pixels). This map depends on the time-varying focal length f of the lenses, unknown *a priori*, but measurable in real time. In the sequel, we concentrate on the pan axis, since the procedure for the tilt axis is similar.

Physical considerations, corroborated by experiments performed while holding the time-varying parameter constant, suggest that the parametric component of the LPV model $\mathcal{F}_u(G_p, \Upsilon)$ can be modeled using just one transfer function, i.e., $p_1\mathcal{F}_u(G_1, \Upsilon)$, and that its dependence with v_1 can be considered to be affine. Regarding the nonparametric component G_{np}, based on the time constant obtained with experiments involving only the mechanical components of the system, we determined a value of $\rho = 1.5$ for the *a priori* stability margin.

The experimental information considered consists of $N = 35$ samples of the time response of the real system y to a unit step input u while the time-varying parameter v_1 was allowed to vary between 0% and 80% of the maximum value of the zoom during the experiment, as is shown in the upper plot of Fig. 3. By repeatedly measuring the location of the centroid of the target in the absence of input, the experimental noise measurement was determined to be bounded by $\epsilon = 4$ pixels, i.e., $\mathcal{N} = \{\boldsymbol{\eta} \in \Re^N\colon |\eta_k| \le \epsilon\}$.

Using these *a priori* information and experimental data, the minimum value of K such that LMIs (3) hold was determined using Matlab's LMI toolbox, yielding a value of $K = 0.0444$, a value of the parameter p_1 of 0.9743 and a nonparametric component with as many states as the number of experimental samples. From a control perspective, minimizing K is attractive since it leads to smaller identification error bounds [through the last term of the upper bound given in (6)] and less conservative designs. The bottom plot of Fig. 3 shows the output of the complete identified model $G_{\mathcal{A}}$, as the sum of a parametric LPV component and a nonparametric LTI component of order 4 (after a Hankel norm model reduction step [4]), to the same input u applied during the experiment and for the measured trajectory of the time-varying parameter v_1, and the noisy measurements of the output of the real system y. As shown there, the identified model explains the observed data within the experimental error.

Regarding the *a priori* worst-case identification error $e(\mathcal{A}, \hat{\mathbf{\Upsilon}})$, the evaluation of (10) leads to an upper bound in the induced ℓ_2-norm



(a)

(b)

Fig. 2. (a) Block diagram of a visual tracking system. (b) Experimental setup.

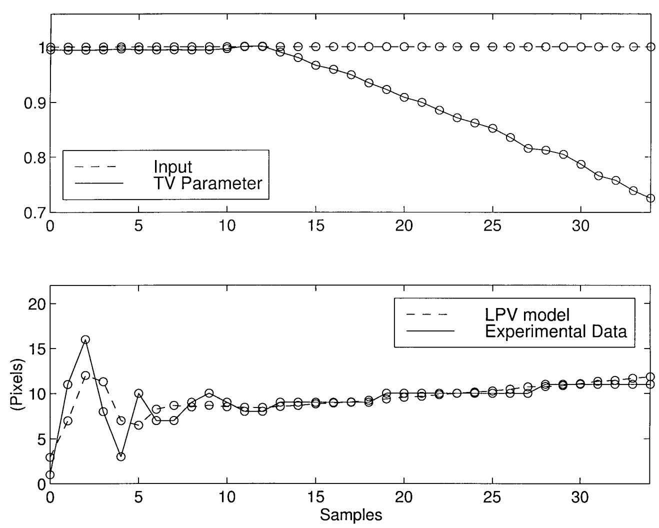


Fig. 3. Results of the identification step.

of 1.15. In order to refine this bound, the LPV model was (in)validated against *new* experimental data of length $n = 35$ corrupted by the same noise level ϵ, subject to additive and multiplicative unstructured uncertainty Δ. For these particular uncertainty types, each LMI $\mathbf{M}^k(\boldsymbol{\zeta})$ of the set (11) depends affinely on δ^2, and therefore it is possible to find the minimum upper bound on the uncertainty norm so that the LPV model is not invalidated. Using this *a priori* information and new experimental data, we determined, using Matlab's LMI toolbox, that the LPV model can explain the new given experimental information, with an LTV uncertainty block bounded in $\|\cdot\|_{\ell_2\,\mathrm{ind}}$ by $\delta_{\mathrm{add}} = 0.018$ and $\delta_{\mathrm{mult}} = 0.26$ in the additive and in the multiplicative cases, respectively.

In order to further validate the proposed approach, the identified model and the uncertainty description $\Delta_{\mathrm{mult}} \in \overline{\mathcal{BL}(\ell_2)}(0.26)$ were combined with the technique used in [2] to design an LFT scheduled $\mathcal{H}_\infty$ controller. As shown in [14], the resulting closed-loop system was able to achieve good tracking performance in spite of the substantial change in the dynamics of the plant due to the change in f.

VI. Conclusion and Directions for Further Research

In this note, we proposed a new robust identification framework that, starting from experimental data, generates models suitable to be used by the LPV synthesis techniques, as well as bounds on the identification error. As shown here, the problems of obtaining and validating a nominal model and an associated uncertainty description are not more computationally demanding that comparable techniques available for the case of LTI systems. Moreover, as in the LTI case, the identification algorithm is optimal up to a factor of 2 as compared with central strongly optimal procedures, and convergent. Efforts are currently under way to extend the LPV model invalidation problem to cope with structured and slowly time-varying uncertainties.

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