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A Solution to MIMO 4–Block l^1 Optimal Control Problems via Convex Optimization

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Abstract

In this paper we propose an alternative solution to 4–block l^1 problems. This alternative is based upon the idea of transforming the l^1 problem into an equivalent (in the sense of having the same solution) mixed $l^1/\mathcal{H}_\infty$ problem that can be solved using convex optimization techniques. The proposed algorithm has the advantage of generating, at each step, an upper bound of the cost that converges uniformly to the optimal cost. Moreover, it allows for easily incorporating frequency and regional pole placement constraints. Finally, it does not require either solving large LP problems or obtaining the zero structure of the plant and computing the so–called zero interpolation and the rank interpolation conditions. The main drawback of this method is that it may suffer from order inflation. However, consistent numerical experience shows that the controllers obtained, albeit of high order, are amenable to model reduction by standard methods, with virtually no loss of performance.

1. Introduction

A large number of control problems involve designing a controller capable of stabilizing a given linear time invariant system while minimizing the worst case response to some exogenous disturbances. This problem is relevant to for instance disturbance rejection, tracking and robustness to model uncertainty (see [8] and references therein). When the exogenous disturbances are modeled as bounded energy signals and performance is measured in terms of the energy of the output, this problem leads to the well known $\mathcal{H}_\infty$ theory. On the other hand, if performance is measured in terms of the peak value of the output, it leads to $\mathcal{H}_2$ theory. Finally, the case where the signals involved are persistent bounded signals, with size measured in terms of the peak time–domain values, leads to the l^1 optimal control theory, formulated by Vidyasagar [8, 9], and solved by Dahleh and Pearson in both the discrete and continuous time cases [1, 2], by using duality to recast the problem into a linear–programming form.

l^1 optimal control theory is appealing because it directly incorporates time–domain specifications. Moreover, it furnishes a complete solution to the robust performance analysis problems [5]. In the SISO and 1–Block (i.e. square) MIMO cases, by exploiting duality theory, the l^1 control problem can be recast into a finite–dimensional optimization problem. In contrast, multiblock MIMO problems do not lead, in general, to finite–dimensional linear programming problems. Rather, at this stage they are solved iteratively, through methods furnishing sequences of upper and lower bounds [3]. In principle, one can attempt to solve the problem by using finite–dimensional approximations. This idea leads to the *Finitely Many Variables (FMV)* method, where the closed–loop system is constrained to be an FIR of some given order, and its dual the *Finitely Many Equations (FME)* where the dual problem is approximated by a finite–dimensional problem. Clearly, the FMV method produces a feasible suboptimal solution yielding an upper bound $\overline{\mu}$ of the optimal cost, while the FME yields an unfeasible super–optimal solution providing a lower bound $\underline{\mu}$ of the cost. A combination of the FMV/FME methods allows for generating a uniform sequence of lower and upper bounds converging to the true optimal, and the optimization stops when the difference between the upper and lower bounds is smaller than a given tolerance. Although this method is easy to implement, its major drawback is that suffers from order inflation, leading to high–order controllers. Moreover, if a low–order optimal controller exists, it may be missed by the method (see [6]).

An elegant alternative to the FMV/FME method is given by the delay augmentation (DA) method, having the advantages of avoiding order inflation (in some cases yielding exact solutions) and providing more insight into the structure of the optimal solutions. Here the idea is to augment the plant with delays, in order to obtain a one–block problem, whose solution can be obtained by using finite–dimensional linear programming. Clearly, the optimal cost for this modified problem provides a lower bound $\underline{\mu}$ of the optimal cost; however, the controller obtained this way is infeasible for the original problem. A feasible controller can

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be recovered by simply discarding the inputs and outputs associated with the delays. This controller yields a cost $\overline{\mu}$ that is an upper bound of the true cost. It can be shown that, under very mild conditions, the lower bound always converges to the true cost. The convergence properties of the upper bound are harder to ascertain. It is shown in [3] that, when the optimal solution is such that the *first* n_u rows of the optimal closed–loop (where n_u is the number of controls) achieve the optimal norm, then $\overline{\mu} \to \mu^o$, the optimal cost. Under this condition, there exists a sequence of optimal closed–loops systems ϕ_N that converges strongly to the optimal solution. It follows that the convergence properties are strongly dependent on the *ordering* of the inputs and the outputs. Thus, a critical step in the optimization is to reorder inputs and outputs in such a way that the set of input–output pairs of minimum order corresponds to the first n_u inputs and outputs. Hence, in general the upper bound $\overline{\mu}$ will not be uniformly non–increasing as N increases.

In this paper we propose an alternative solution to 4–block l^1 problems. This alternative is based upon the idea of transforming the l^1 problem into an equivalent (in the sense of having the same solution) mixed $l^1/\mathcal{H}_\infty$ problem. Using the methods in [7] this latter problem can be solved by solving a sequence of problems, each one consisting of a constrained convex optimization problem and an unconstrained $\mathcal{H}_\infty$ problem. The proposed algorithm has the advantage of generating, at each step, an upper bound of the cost that converges uniformly to the optimal cost. Moreover, it allows for easily incorporating frequency and regional pole placement constraints. Finally, it does not require either solving large LP problems or obtaining the zero structure of the plant and computing the so–called zero interpolation and the rank interpolation conditions. The main drawback of the method is that it may suffer from order inflation. However, consistent numerical experience shows that the controllers obtained, albeit of high order, are amenable to model reduction by standard methods, with virtually no loss of performance.

The paper is organized as follows: In section 2 we introduce the notation to be used and some preliminary results. In section 3 we show that, when suitably modified, the l^1 problem is equivalent to a mixed $l^1/\mathcal{H}_\infty$ problem, that in the limit solves the original, unmodified problem. Here, we also recall the main result of [7], showing that this modified problem can be reduced to a finite–dimensional convex optimization and an unconstrained $\mathcal{H}_\infty$ problem. The applicability of the method is illustrated in section 4 with a simple design example. Finally, in section 5, we summarize our results and we present some concluding remarks.

2. Preliminaries

2.1. Notation and Preliminary Results

Given a matrix A, we denote by A_i its i-th row. For $x \in R^n$ we define $|x|$ as the vector with components $|x_i|$. We denote the 1−norm as $\|x\|_1 \stackrel{\Delta}{=} \sum_{i=0}^{n} |x_i|$ and the infinity norm as $\|x\|_\infty \stackrel{\Delta}{=} \max_i |x_i|$. l_1 denotes the space of absolutely summable sequences $h = \{h_i\}$ equipped with the norm $\|h\|_1 \stackrel{\Delta}{=} \sum_{i=0}^{\infty} |h_i| < \infty$. l_∞ denotes the space of bounded sequences $h = \{h_i\}$ equipped with the norm $\|h\|_\infty \doteq \sup_{i \geq 0} |h_i| < \infty$. We denote by l_∞^p the space of bounded vector sequences $\{h(k) \in R^p\}$. In this space we define the norm $\|h\|_\infty \doteq \sup_i \|h_i(k)\|_\infty$. Assume now that $H : l_\infty^q \to l_\infty^p$ is a bounded linear operator defined by the usual convolution relation $y = H * u$. If we denote by $H(k)$ the Markov parameters of H, its induced $l_\infty^q \to l_\infty^p$ norm is given by:

$$\|H\|_1 \doteq \max_i \sum_{j=1}^{n} \|h_{ij}\|_1 \ = \ \max_i \sum_{k=0}^{\infty} \|h_i(k)\|_1$$

$\mathcal{L}_\infty$ denotes the Lebesgue space of complex valued matrix functions which are essentially bounded on the unit circle. $\mathcal{H}_\infty$ denotes the space of transfer matrices $G(z) \in \mathcal{L}_\infty$ which are analytic outside the unit disk, equipped with the norm $\|G(z)\|_\infty \doteq \sup_{0 \leq \theta \leq \pi} \overline{\sigma}\left(G(e^{j\theta})\right)$, where $\overline{\sigma}$ denotes the largest singular value.

Next, we recall two well known properties relating the $\mathcal{H}_\infty$ and l_1 norm of a stable, finite–dimensional, linear time–invariant (FDLTI) system:

Lemma 1 *Let G be a $p \times m$ stable FDLTI system, with McMillan degree n. Then:*

1. $\|G\|_\infty \leq \sqrt{m}\|G\|_1$
2. $\|G\|_1 \leq (2n+1)\sqrt{m}\|G\|_\infty$

Proof: see [4], Chapter 4.

2.2. The MIMO l^1 Optimal Control Problems

By using the YJBK parametrization of all stabilizing controllers [1][10], the MIMO discrete–time l^1 control problem can be recast into the following norm minimization form:

Problem 1

$$\mu_o = \inf_{Q \in l_1^{n_u \times n_y}} \|T_{11} - T_{12} Q T_{21}\|_1 \qquad (1)$$

where $T_{11} \in l_1^{n_z \times n_w}$, $T_{12} \in l_1^{n_z \times n_u}$ and $T_{21} \in l_1^{n_y \times n_w}$, and where n_y, n_w, n_z and n_u are the dimensions of the output available to the controller, the exogenous disturbance, the performance variable and the control input respectively (see [3] for details). Existence of a solution is guaranteed if T_{12} and T_{21} do not have zeros on the unit circle.

Remark 1 *By using duality theory, problem (1) can be recast into a linear programming problem. However, in contrast to the 1–block case, in the 4–block case both the primal and the dual problems have an infinite number of variables and constraints. The reader is referred to [3] for an extensive treatment of multi-block problems and in particular the conditions under which these problems admit an exact solution with finite–support.*

3. Main Results

In this section we propose a method, based upon convex optimization, for solving problem (1). The main idea of the method is to recast (1) into an equivalent mixed $l^1/\mathcal{H}_\infty$ problem, which can be solved by exploiting the results in [7].

Lemma 2 *Assume that* $\mu_o \doteq \inf_{Q \in l_1^{n_u \times n_y}} \|T_{11} - T_{12}QT_{21}\|_1 \leq \gamma$. *Then the l^1 optimal control problem (1) is equivalent to the following mixed $l^1/\mathcal{H}_\infty$ problem:*

Problem 2 *Find:*

$$\begin{aligned} \mu_o &= \inf_{Q \in l_1^{n_u \times n_y}} \|T_{11} - T_{12}QT_{21}\|_1 \\ \text{subject to} & \\ & \|T_{11} - T_{12}QT_{21}\|_\infty \leq \sqrt{n_z}\gamma \end{aligned} \tag{2}$$

Proof: The proof follows immediately from Lemma 1 by noting that at the optimum the $\mathcal{H}_\infty$ constraint is inactive.

Remark 2 *Lemma 2 states that if an upper bound of the optimal l_1 cost is available, it can be used to transform problem (1) into (2). This bound can be obtained, for instance, by finding the optimal $\mathcal{H}_\infty$ controller for the plant and then using the second part of Lemma 1. Alternatively, this upper bound can be obtained by using the FMV method.*

Next we recall the main result of [7] showing that problem (2) can be solved by solving a sequence of problems. Each one entails solving a finite–dimensional constrained convex optimization and a standard $\mathcal{H}_\infty$ problem. In this section we briefly review this result, established by showing that: i) $(l^1/\mathcal{H}_\infty)$ can be solved by considering a sequence of modified problems; ii) Given $\epsilon > 0$, an ϵ–suboptimal solution to the modified problem can be found by solving a truncated problem; and iii) this truncated problem can be decoupled into a finite–dimensional constrained convex optimization and an unconstrained $\mathcal{H}_\infty$ problem.

Let $\mathcal{RH}_{\infty,\delta}$ denote the subspace of transfer matrices in $\mathcal{RH}_\infty$ which are analytic outside the disk of radius δ, $0 < \delta < 1$, equipped with the norm $\|G(z)\|_{\infty,\delta} \doteq \sup_{0 \leq \theta \leq \pi} \overline{\sigma}\left(G(\delta e^{j\theta})\right)$, and consider the following modified problem:

Problem 3 *$(l^1/\mathcal{H}_{\infty,\delta})$*
Given $\gamma > \gamma^* \doteq \inf_{Q \in \mathcal{H}_{\infty,\delta}} \|T_{11} - T_{12}QT_{21}\|_{\infty,\delta}$, *and* $T_{11}(z)$, $T_{12}(z)$, $T_{21}(z) \in \mathcal{RH}_{\infty,\delta}$, *find:*

$$\mu_\delta = \inf_{Q \in \mathcal{H}_{\infty,\delta}} \|T_{11} - T_{12}QT_{21}\|_1 \tag{3}$$

subject to:

$$\|T_{11} - T_{12}QT_{21}\|_{\infty,\delta} \leq \gamma$$

where $0 < \delta < 1$.

Remark 1 *Without loss of generality, it can be assumed that T_{12} and T_{21} are respectively inner and co–inner in $\mathcal{RH}_{\infty,\delta}$ (i.e. $T'_{12}(\frac{1}{\delta z})\ T_{12}(\delta z) = I$ and $T_{21}(\delta z)\ T'_{21}(\frac{1}{\delta z}) = I$, where $'$ indicates transpose). Moreover, if $T_{12}(T_{21})$ is not square, it is possible to choose $T_{12\perp}(T_{21\perp})$ such that $T_{12a} = [T_{12\perp}\ \ T_{12}]$ $(T'_{21a} = [T'_{21\perp}\ \ T'_{21}]')$ is unitary in $\mathcal{RH}_{\infty,\delta}$. This can be accomplished by using the change of variable $z = \delta\overline{z}$ coupled with a standard inner–outer factorization (see [7] for details).*

Remark 2 *From the maximum modulus theorem, $\|T_{11} - T_{12}QT_{21}\|_\infty \leq \|T_{11} - T_{12}QT_{21}\|_{\infty,\delta}$. It follows that any feasible transfer matrix for Problem 2 is also feasible for Problem 3. Since both problems have the same objective function, it follows that μ_δ is an upper bound for μ_o. The same reasoning also shows that $\mu_{\delta_1} \geq \mu_{\delta_2}$ whenever $\delta_1 < \delta_2$.*

Lemma 3 *Consider an increasing sequence $\delta_i \to 1$. Let μ_o and μ_{δ_i} denote the solution to problems (2) and (3) respectively. Then the sequence $\mu_{\delta_i} \to \mu_o$.*

Proof: see Lemma 1 in [7].

Theorem 1 *Let* $G = T_{21a} T_{11}^{'} T_{12a}$, *where* $T_{11}^{'} = T_{11}^{'}(\frac{1}{\delta z})$. *Suppose G has a state space realization:*

$$G = \left(\begin{array}{c|cc} \hat{A} & B_a & B_b \\ \hline C_a & D_{aa} & D_{ab} \\ C_b & D_{ba} & D_{bb} \end{array} \right)$$

Then, a suboptimal solution to the mixed $l^1/\mathcal{H}_{\infty,\delta}$ *control problem, with cost* μ_δ^ϵ, $\mu_\delta \leq \mu_\delta^\epsilon \leq \mu_\delta + \epsilon$, *is given by* $Q^o = Q_F^o + z^{-N} Q_R^o$ *,where* $Q_F^o = \sum_{i=0}^{N-1} Q_i z^{-i}$;$\underline{Q} = \begin{bmatrix} Q_o & 0 \cdots 0 \\ Q_1 & Q_o \cdots 0 \\ \vdots & \ddots \\ Q_{N-1} \cdots & \cdots Q_o \end{bmatrix}$ *solves the following finite dimensional convex optimization problem:*

$$\underline{Q} = \underset{\substack{\underline{Q} \in R^{n_u \cdot N n_y} \\ \|\mathcal{Q}\|_2 \leq \gamma}}{\operatorname{argmin}} \|\underline{t}_1 + \mathcal{T}_{12} \underline{Q} \mathcal{T}_{21}\|_1$$

and $Q_R(z)$ *solves the approximation problem*

$$Q_R^o = \underset{Q_R \in \mathcal{RH}_{\infty,\delta}}{\operatorname{argmin}} \|T_{11}(z) + T_{12}(z) Q_F^o T_{21}(z) + z^{-N} T_{12}(z) Q_R T_{21}(z)\|_{\infty,\delta}$$

where:

$$\mathcal{Q}(\underline{Q}) =$$

$$\begin{bmatrix} y\hat{A}^N x & y\hat{A}^{N-1}B_a & \cdots y\hat{A}B_a & yB_a & y\hat{A}^{N-1}B_b & y\hat{A}^{N-2}B_b & \cdots y\hat{A}B_b & yB_b \\ C_a\hat{A}^{N-1}x & C_a\hat{A}^{N-2}B_a & \cdots C_aB_a & D_{aa} & C_a\hat{A}^{N-2}B_b & C_a\hat{A}^{N-3}B_b & \cdots C_aB_b & D_{ab} \\ C_a\hat{A}^{N-2}x & C_a\hat{A}^{N-3}B_a & \cdots D_{aa} & 0 & C_a\hat{A}^{N-3}B_b & C_a\hat{A}^{N-4}B_b & \cdots D_{ab} & 0 \\ \vdots & \vdots & \vdots \ddots & \vdots & \vdots & \vdots & \ddots \vdots & \vdots \\ C_a x & D_{aa} & 0 \cdots & 0 & D_{ab} & 0 & 0 \cdots & 0 \\ C_b\hat{A}^{N-1}x & C_b\hat{A}^{N-2}B_a & \cdots C_bB_a & D_{ba} & C_b\hat{A}^{N-2}B_b & C_b\hat{A}^{N-3}B_b & \cdots C_bB_b & Q_0^T \\ C_b\hat{A}^{N-2}x & C_b\hat{A}^{N-3}B_a & \cdots D_{ba} & 0 & C_b\hat{A}^{N-3}B_b & C_b\hat{A}^{N-4}B_b & \cdots Q_0^T & Q_1^T \\ \vdots & \vdots & \vdots \ddots & \vdots & \vdots & \vdots & \ddots \vdots & \vdots \\ C_b x & D_{ba} & 0 \cdots & 0 & Q_0^T & Q_1^T & \cdots \cdots & Q_{N-1}^T \end{bmatrix}$$

$$\begin{aligned} \underline{t}_1 &= \begin{bmatrix} T_{11}{}'_o & \cdots & T_{11}{}'_{N-1} \end{bmatrix}' \\ \mathcal{T}_{12} &= \begin{bmatrix} T_{12_o} & 0 & \cdots & 0 \\ T_{12_1} & T_{12_o} & \cdots & 0 \\ \vdots & & \ddots & \\ T_{12_{N-1}} & \cdots & & T_{12_o} \end{bmatrix} \\ \mathcal{T}_{21} &= \begin{bmatrix} T_{21_o} \\ T_{21_1} \\ \vdots \\ T_{21_{N-1}} \end{bmatrix} \\ N &= \left\lceil \frac{\log \epsilon(1-\delta) - \log M}{\log \delta} \right\rceil \\ M &= \sqrt{n_w n_z}\gamma \\ x &= X^{\frac{1}{2}} \\ y &= Y^{\frac{1}{2}} \end{aligned} \tag{4}$$

where Q_k, T_{ij_k} *denote the* k^{th} *element of the impulse responses of* $Q(z), T_{ij}(z)$ *respectively,* $X > 0$ *and* $Y > 0$ *are the solutions to following Riccati equations:*

$$\begin{aligned} \hat{X} &= \left(\hat{A}\hat{X}C_a^T + B_e D_{er}^T\right)\left(\gamma^2 I - D_{er}D_{er}^T - C_a\hat{X}C_a^T\right)^{-1}\left(C_a\hat{X}\hat{A}^T + D_{er}B_e^T\right) \\ &\quad + \hat{A}\hat{X}\hat{A}^T + B_e B_e^T \\ \hat{Y} &= \left(\hat{A}^T\hat{Y}B_a + C_e^T D_{ec}\right)\left(\gamma^2 I - D_{ec}^T Dec - B_a^T\hat{Y}B_a\right)^{-1}\left(B_a^T\hat{Y}\hat{A} + D_{ec}^T C_a\right) \\ &\quad + \hat{A}^T\hat{Y}\hat{A} + C_e^T C_e \end{aligned} \tag{5}$$

and where, for notational simplicity, we defined:

$$\begin{aligned} B_e &= \begin{bmatrix} B_a & B_b \end{bmatrix} \\ C_e &= \begin{bmatrix} C_a \\ C_b \end{bmatrix} \\ D_{er} &= \begin{bmatrix} D_{aa} & D_{ab} \end{bmatrix} \\ D_{ec} &= \begin{bmatrix} D_{aa} \\ D_{ba} \end{bmatrix} \end{aligned} \tag{6}$$

3.1. Solution to MIMO l^1 Problems

Combining the results of Lemmas 2, 3 and Theorem 1, it follows that a solution to MIMO 4–block l^1 problems can be obtained using the following algorithm:

1. Data: a sequence $\delta_i \to 1$, $\delta_i < 1$.
2. Find an upper bound γ_i such that $\inf_{Q \in \mathcal{RH}_{\infty,\delta}} \|T_{11} + T_{12}QT_{21}\|_{\infty,\delta} \leq \gamma_i$. This upper bound can be found using for instance the change of variable $z \to \delta z$ and the FMV method (or standard $\mathcal{H}_\infty$ methods).
3. Solve the mixed $l^1/\mathcal{H}_{\infty,\delta}$ problem using Theorem 1
4. Repeat until the l_1 cost is sufficiently close to a lower bound (obtained for instance using the FME or DA methods).

It is clear that at each stage the algorithm produces an upper bound of the true optimal l_1 cost. Note that the $\mathcal{H}_\infty$ constraint subsumes information on the behavior of the objective function after the horizon N, rather than just truncating the objective (as in the FMV method).

Also, note that, since the bound γ_i is chosen at each stage large enough as not to affect the l_1 cost, the only difference between the original and modified l^1 problems is that the latter is constrained to have *all* the poles of the closed–loop system constrained to the interior of the δ–disk. Thus, if this additional constraint is added to the original l^1 problem, the algorithm yields the *exact* solution, without the need of iterations. From an engineering stand–point, it can be argued that this additional constraint is desirable, since by inducing a rate of decay faster than δ it avoids closed–loop systems with long settling times.

4. A Simple Example

Consider the pitch axis control of a forward–swept wing X29 aircraft [3]. A simplified model of the plant is given by:

$$P(s) = \frac{(s+3)}{(s+10)(s-6)} \frac{20}{(s+20)} \frac{(s-26)}{(s+26)}$$

The objective is to design a compensator K to minimize $\left\| \begin{matrix} W_1 K S \\ W_2 S \end{matrix} \right\|_1$ where S denotes the sensitivity function and the weights are chosen as $W_1 = 0.01$ and $W_2 = \frac{(s+1)}{(s+.001)}$ (see [3] for details). In order to obtain a discrete–time problem, the plant is discretized via a zero–order hold at the inputs and sampling at $T_s = \frac{1}{30}$. Constraining all the closed–loop to the interior of the $\delta = 0.967$ disk yields $N = 100$ and $\left\| \begin{matrix} W_1 K S \\ W_2 S \end{matrix} \right\|_1 = 4.08$. Model reduction yields the following 6^{th} order controller (with the same performance):

$$K(z) = \frac{-73.8738z^6+32.3619z^5+134.8z^4-48.0132z^3-115.146z^2+85.8354z-16.3459}{z^6 + 0.0522z^5 - 0.3341z^4 - 1.1922z^3 - 0.6960z^2 + 0.7766z + 0.3936}$$

The frequency response of the controller is shown in Figure 1. It is interesting to compare this controller with the controller obtained in [4] using the DA method. Both controllers have the same order and similar frequency responses. The controller in [3] achieves a higher l_1 cost (4.1) while placing a closed–loop pole within 10^{-5} of the unit circle. Thus, it can potentially lead to large settling times. On the other hand, the controller obtained using our procedure places all the poles within the $\delta = 0.967$ disk, and lower cost is achieved. Note also in passing that the large value of N in this example is partially due to the open loop pole very close to $z = 1$, due to the choice of weighting functions.

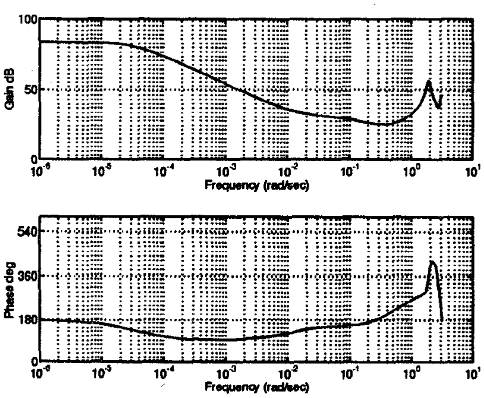

Figure 1: frequency response of the controller

5. Conclusion

In this paper we propose an alternative solution to 4–block l^1 problems. This alternative is based upon the idea of transforming the l^1 problem into an equivalent (in the sense of having the same solution) mixed $l^1/\mathcal{H}_\infty$ problem. Using the methods in [7] this latter problem can be solved by solving a sequence of problems, each one consisting of a finite–dimensional constrained convex optimization problem and an unconstrained $\mathcal{H}_\infty$ problem. The proposed algorithm has the advantage of generating, at each step, an upper bound of the cost that converges uniformly to the optimal cost. Moreover, it allows for easily incorporating frequency and regional pole placement constraints. Finally, it requires neither solving large LP problems nor obtaining the zero structure of T_{12} and T_{21} and computing the so–called zero interpolation and the rank interpolation conditions. The main drawback of the method is the fact that it may suffer from order inflation. However, consistent numerical experience shows that the controllers obtained, albeit of high order, are amenable to model reduction by standard methods, with virtually no loss of performance.

We believe that the proposed method provides a useful alternative to Delay Augmentation, specially for cases where the number of inputs or outputs is not small. In these cases, DA will tend to result in larger LP problems, and it may require a large number of trial and error type iterations (reordering inputs and outputs) in order to satisfy the sufficient conditions for convergence of the upper bound.

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