A Carathéodory-Fejér Approach to Simultaneous Fault Detection and Isolation

Duk-Sun Shim
School of Electrical and Electronic Engineering
Chung-Ang University
221 Huksuk-dong, Dongjak-ku
Seoul, Korea 156-756
email: dshim@cau.ac.kr

Mario Sznaier
Department of Electrical Engineering
The Pennsylvania State University
University Park, PA 16802
email: msznaier@gandalf.ee.psu.edu

Abstract

In this paper we address the problem of detecting and isolating faults from noisy input/output measurements of a MIMO uncertain–system. The main result of the paper shows that this problem can be solved by using Carathéodory-Fejér's theorem to reduce it to an LMI feasibility problem. These results are illustrated with a simple example.

1 Introduction

The problem of Fault Detection and Isolation (FDI) in automated processes and control systems has been the subject of considerable attention during the past two decades. This research has resulted in a variety of methods and a vast amount of papers in the literature (see for instance [4, 6, 10] and references therein). Many of these methods are based on a *model–based* approach, also known as analytical or functional redundancy. In contrast to approaches based on *physical or hardware redundancy*, the former exploit the mathematical model of the system under consideration, leading to a two stage procedure: (i) residual generation and, (ii) decision making.

While appealing, since does not require additional hardware, a potential problem with the analytical approach is its *fragility*: a mismatch between the actual plant and the model used in the FDI algorithm can result in false alarms. To avoid this difficulty, the algorithm must be robust both against modelling errors and exogenous disturbances. Robust FDI methods have been well researched (see for instance [10] and references therein), but only relatively few papers address robustness issues in the context of model–based approaches [2, 7, 8, 9, 12]). An potential disadvantage of these methods is the difficulty in isolating the exact location of the fault and in detecting simultaneous faults.

In this paper we address the problem of robust analytical FDI for systems subject to parametric dynamic faults, in the presence of multiplicative dynamic uncertainty and exogenous disturbances. The main result of the paper shows that by using Carathéodory-Fejér's theorem [3], the problem can be reduced to an LMI feasibility problem and efficiently solved. The proposed approach can detect and isolate simultaneous faults and does not necessitate separate residual generation and decision steps.

The paper is organized as follows. In section 2, we introduce some preliminary results. Section 3 contains a precise problem statement and the proposed solution. Finally, these results are illustrated in section 4 with a simple example.

2 Preliminaries

2.1 Notation

In the sequel, ℓ_2^q denotes the Hilbert space of square summable vector sequences:

$$\begin{aligned} \ell_2^q &\doteq \{\mathbf{x} = (x(0), x(1), \cdots) : x(i) \in R^q, \\ ||\mathbf{x}||^2 &= \textstyle\sum_{i=0}^{\infty} ||x(i)||^2 < \infty\}. \end{aligned}$$

For a vector sequence $\mathbf{x} = \{x(0), x(1), \cdots, x(l-1) \in R^q\} \in \ell_2^q$, $T_x \in R^{lq \times l}$ denotes its associated lower block Toeplitz matrix:

$$T_x = \begin{bmatrix} x(0) & 0 & 0 & \cdots & 0 \\ x(1) & x(0) & 0 & \cdots & 0 \\ x(2) & x(1) & x(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ x(l-1) & x(l-2) & x(l-3) & \cdots & x(0) \end{bmatrix}.$$

Similarly, given a causal, linear time-invariant $m \times n$ matrix function $G(s)$, $T_G \in R^{ml \times nl}$ denotes the lower block Toeplitz matrix associated with its impulse response sequence $(G(0), G(1), \cdots)$,i.e.

$$T_G = \begin{bmatrix} G(0) & 0 & 0 & \cdots & 0 \\ G(1) & G(0) & 0 & \cdots & 0 \\ G(2) & G(1) & G(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ G(l-1) & G(l-2) & G(l-3) & \cdots & G(0) \end{bmatrix}.$$

2.2 Background results

In this section we recall, for ease of reference, some results on the existence of a bounded ℓ^2 operator mapping two given sequences. These results will be used in the sequel to recast the FDI problem into an LMI feasibility form.

Lemma 1 (Carathéodory-Fejér) *[3, 11, 1]. Given two sequences* $\mathbf{u} = \{u(0), u(1), \cdots, u(l-1) \in R^n\}$ *and* $\mathbf{y} = \{y(0), y(1), \cdots, y(l-1) \in R^m\}$, *there exists a stable, causal, linear time-invariant operator with* $\| \ \|_\infty \leq \gamma$ *such that*

$$\mathrm{u} = \mathrm{y}$$

if and only if $T_y' T_y \leq \gamma^2 T_u' T_u$.

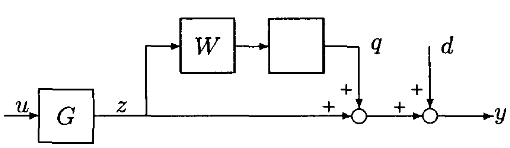

Figure 1: Setup for Robust Analytic FDI.

Corollary 1 *Consider the block diagram shown in Figure 1 where the known transfer matrices G and W represent the nominal plant and uncertainty weights respectively, represents LTI, causal, ℓ^2- bounded dynamic uncertainty, and where d represents an unknown but ℓ^2 bounded disturbance. Then given experimental input output measurements u and y, there exists:*

$$\in \quad = \quad \{ \quad : LTI, \| \quad \|_\infty < \gamma\} \qquad (1)$$

$$\mathrm{d} \in \mathcal{D} \quad = \quad \{\boldsymbol{d} = (d_0, d_1, \cdots, d_{l-1}) : \quad d_i \in R^m \text{ and } \sum_{i=0}^{l-1} d_i' d_i \leq \alpha\} \qquad (2)$$

such that

$$y = (I + \ \tilde{} W) G u + d \qquad (3)$$

if and only if there exists $d \in \mathcal{D}$ such that the following inequality is satisfied:

$$(T_y - T_d - T_G T_u)' (T_y - T_d - T_G T_u) - \gamma^2 T_u' T_G' T_W' T_W T_G T_u < 0 \qquad (4)$$

The corollary above, combined with Schur complements, is the key result in reducing a FDI problem to an LMI feasibility form[1].

3 Robust Fault Detection and Isolation

3.1 Problem Formulation

In this paper we consider the problem of fault detection and isolation for systems represented by the following parametrized fault model which includes both multiplicative dynamic uncertainty and disturbances:

$$y = (I + \quad W)(G_0 + \sum_{i=1}^{r} f_i G_i) u + d \qquad (5)$$

Here the transfer matrices G_0 and $G_i, i = 1, \cdots, r$ represent the nominal plant and dynamic fault models, respectively, the scalars f_i are fault indicators, and d represents an unknown but ℓ^2 bounded disturbance. In the sequel we address the following two cases:

1. *Hard faults:* This case models the situation where only r discrete fault modes can appear in the system. The corresponding indicator f_i can take only binary values, with $f_i = 1$ indicating the presence of the i^{th} failure mode.

2. *Continuous fault case:* In this case faults can appear gradually, a situation modelled by allowing $f_i \in [0, 1]$, with $f_i = 1$ corresponding to the extreme case of total failure. Here the goal is not only to determine whether the i^{th} failure mode is present, but also to estimate its strength.

[1] see also [11, 1] for an application to the problem of model (in)validation.

In this context, the FDI problem can be stated as follows:

Problem 1 *Given the nominal model (G_o, W), failure dynamics G_i, uncertainty sets , $\mathcal{D}$, and N_m input/output experimental measurements determine:*

1. *whether a fault has occurred,*
2. *in that case isolate it, and, in the continuous fault case, determine its strength.*

In the sequel, for notational simplicity, we consider the SISO case. However, the derivations are completely general and can be extended to the MIMO case at the price of a more involved notation.

3.2 Problem solution.

The following theorem provides a necessary and sufficient condition for fault detection and isolation in plants having a parametrized fault model of the form (5).

Theorem 1 *Consider the system model (5). Then, given experimental input/output data $\{u_0^{N_m-1}\}$, $\{y_0^{N_m-1}\}$, there exist $\in$, $\mathrm{d} \in \mathcal{D}$, and a combination of faults that explains this experimental outcome if and only the corresponding f_i satisfy the the following set of LMIs*

$$\begin{bmatrix} X & (T_y - T_d)' \\ (T_y - T_d) & -I \end{bmatrix} < 0$$
$$\begin{bmatrix} -\alpha^2 & \mathbf{d}' \\ \mathbf{d} & -I \end{bmatrix} \leq 0 \qquad (6)$$

where

$$\begin{aligned} X &= -(T_y - T_d)'T_G T_u - T_u' T_G' (T_y - T_d) \\ &+ T_u' T_G' (I - \gamma^2 T_W' T_W) T_G T_u \end{aligned}$$

for some vector $\mathrm{d} \in R^{N_m}$, *where*

$$T_G = T_{G_0} + \sum_{i=1}^{r} f_i T_{G_i}$$

Proof: Consider the signals $z = (G_0 + \sum_{i=1}^r f_i G_i)u$ and $q = \ \ Wz$ shown in Figure 1. From Lemma 1, combined with the fact that $y = z + q + d$ it follows that $\in$ if and only if the following inequality holds:

$$T_q' T_q - \gamma^2 T_z' T_W' T_W T_z < 0$$

The proof follows by substituting $q = y - d - z$ into the above inequality and using the fact that $T_z = T_G T_u$, combined with a Schur complement argument.

Corollary 2 *Given the experimental input/output data $\{u_0^{N_m-1}\}, \{y_0^{N_m-1}\}$, if the set of LMIs (6) is infeasible with $f_i = 0$, $i = 1 \ldots r-1$, then a fault has occurred.*

The corollary above gives a sufficient condition for detecting faults[2]. Next, we indicate how to use these results also to isolate the location of the fault(s).

1. *Hard fault case:* Recall that in this case $f_i \in \{0, 1\}$. Thus, once existence of a fault has been established, candidate sets of faults can be obtained by determining which of the 2^r vertices of the hypercube $\|f\|_\infty = 1$ lead to feasibility of the LMIs (6). Note that, due to the existence of uncertainty and noise, this approach can yield more than one candidate fault. In this case, one could try to find the most "likely" failure mode by finding, for each of these candidates, the one associated with the lowest model uncertainty norm, γ (or alternatively, lowest noise level α). Since the f_i are fixed, optimizing over γ^2 or α^2 subject to feasibility of (6) is a convex problem.

2. *Continuous fault case:* The goal here is twofold: to identify the faults and determine their extent. A potential problem here is that the inequality (6) is *bilinear* in f_i and d. Therefore, in the continuous time case, while noisy output measurements of an uncertain system allow for detecting that a fault is present, they may not allow for isolating it. To proceed further, we need to consider either the noiseless measurements case (d = 0) or the completely known dynamics case (= 0).

   (a) *noiseless measurements:* In this case, straightforward Schur complement arguments applied to the first inequality in (6) show that the candidate faults are those that satisfy the following condition:

   $${}_0 + \sum_{i=1}^{r} f_i \ {}_i < 0 \qquad (7)$$

   where

   $${}_0 = \begin{bmatrix} X & T_u' T_{G_0}' \\ T_{G_0} T_u & -(I - \gamma^2 T_W' T_W)^{-1} \end{bmatrix},$$

   $$X = T_y' T_y - T_y' T_{G_0} T_u - T_u' T_{G_0}' T_y$$

[2] the condition is not necessary since some faults can be masked by the presence of noise and model uncertainty. In the deterministic, noiseless case, e.g. $\gamma = \alpha = 0$, the condition becomes indeed necessary and sufficient.

and

$$_i = \begin{bmatrix} -T_y' T_{G_i} T_u - T_u' T_{G_i}' T_y & T_u' T_{G_i}' \\ T_{G_i} T_u & 0 \end{bmatrix},$$

(b) *no uncertainty case:* In this case, the second LMI in (6), combined with the fact that $d = y - T_G u$ leads to

$$\begin{bmatrix} -\alpha^2 & X' \\ X & -I \end{bmatrix} < 0 \qquad (8)$$

$$X = y - u\,(T_{G_0} + \sum f_i T_{G_i})$$

an LMI in the variables f_i, α^2.

As before, there may be multiple combinations of f_i that render the LMIs (7) or (8) feasible. In this case, one may attempt to select one by minimizing either α^2, a convex function of f such as $\|f\|$, or γ^2. The first two approaches lead to a convex LMI problem, while the third one can be solved by performing a one–dimensional line search.

4 Illustrative Example

In this section we illustrate the potential of the proposed approach using a simplified model of the yaw damper system of a jet transport. The system under consideration is given by

$$y = (I + \;\; W)(G_0 + \sum_{i=1}^{3} f_i G_i)u + d \qquad (9)$$

where

$$G_0 = \frac{1}{D_0(s)} \begin{bmatrix} -4.75s^3 - 2.48s^2 & 1.23s^3 + 0.30s^2 \\ -1.19s - 0.56 & +0.83s + 0.42 \\ 1.15s^2 - 2.00s & 10.73s^2 \\ -13.73 & +16.43s + 10.83 \end{bmatrix}$$

$$G_1 = \frac{1}{D(s)} \begin{bmatrix} 6.08s^6 + 4.69s^4 & -1.57s^6 - 1.65s^4 \\ +1.13s^2 & -0.59s^2 \\ 1.80s^4 + 9.63s^2 & -0.57s^4 - 1.43s^2 \end{bmatrix}$$

$$G_2 = \frac{1}{D(s)} \begin{bmatrix} 6.35s^5 + 0.36s & -1.21s^5 - 0.26s \\ -1.47s^5 + 4.69s & 0.38s^5 - 3.27s \end{bmatrix}$$

$$G_3 = \frac{1}{D(s)} \begin{bmatrix} 3.03s^3 + 0.086 & -1.37s^3 - 0.06 \\ 18.54s^3 + 2.08 & -4.46s^3 - 1.55 \end{bmatrix}$$

and

$$D_0(s) = s^4 + 1.92s^3 + 1.61s^2 + 0.83s + 0.16$$

$$D(s) = s^8 + 2.55s^7 + 3.76s^6 + 4.16s^5 + 3.18s^4 + 1.71s^3 + 0.58s^2 + 0.0826s + 0.0006$$

Here the inputs to the system are rudder and aileron deflections, measured in degrees, and the outputs are yaw rate and bank angle.

Assume that the only a priori information available about the model uncertainty and noise is that $\| \;\; \|_\infty = 0.02$ and $\alpha = 0.0001$. Thus we can take $T_W = I$. Further, assume that the *actual* plant model is given by $G_{actual} = (I + \tilde{\;})G$, with

$$\tilde{\;} = \frac{1}{s^2 + 12s + 32} \begin{bmatrix} 0.048s + 0.02 & 0.01s + 0.01 \\ 0.082s + 0.01 & 0.081s + 0.02 \end{bmatrix}$$

for which $\| \tilde{\;} \|_\infty = 0.013$.

Next we consider two different scenarios:

1. *Discrete Fault Case:* Since in this case the parametrized model (9) has three independent fault dynamics, there are in total eight different combinations, ranging from no fault (corresponding to $(f_3 f_2 f_1) = (0, 0, 0)$) to a failure in each mode (e.g. $(f_3 f_2 f_1) = (1, 1, 1)$).

   Table 1 shows the results of 8 experiments, corresponding to each of these cases. In all cases, we generated 21 samples of the impulse response of $(I + \tilde{\;})G_f$[3], corrupted by noise $\tilde{d}$, and checked the corresponding LMIs for feasibility, by minimizing t subject to $\mathcal{L}(\mathrm{f}, \tilde{d}) < tI$, where $\mathcal{L}(.)$ denotes the left–hand side in (6). Each row in Table 1 corresponds to a single experiment, with the entries indicating the minimum value of t for the corresponding combination of f_i. For instance, the second entry in the first row corresponds to the actual system without faults ($f_1 = f_2 = f_3 = 0$), checked against the hypothesis $f_1 = 1, f_2 = 0, f_3 = 0$. As shown there, the only feasible set of LMIs (negative entries) corresponds to entries in the diagonal, that is, in each case the algorithm successfully identified and isolated the faults.

2. *Continuous Fault Case:* Assume now that the faults f_1, f_2, and f_3 are continuous variables in [0,1]. As discussed before, in order to isolate faults in this case, we need to consider noise–free measurements. As before, we considered 21 data points of the impulse response of the system $(I + \tilde{\;})G_f$. In this case, the estimate corresponding to the case where no faults are present (e.g. $f = [0\ 0\ 0]$) is $\hat{f} = [0.0035\ 0.0025\ 0.0005]$, indicating that the experimental data indeed can

[3] Here G_f denotes the transfer function corresponding to the failure mode under consideration.

Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

candidate test fault

$f_3f_2f_1$	000	001	010	011	100	101	110	111
0	**-0.0000**	2.6866	1.4478	5.2696	1.6711	2.5513	4.4538	1.8367
1	2.6418	**-0.0000**	1.4679	7.9932	0.8594	3.1155	6.2482	2.3081
2	2.7019	3.7169	**-0.0001**	3.3179	1.2222	3.4023	2.1372	1.0255
3	11.7652	8.4483	7.9818	**-0.0001**	6.9802	0.7109	0.6548	0.9331
4	8.1555	2.5482	1.0106	7.6870	**-0.0000**	3.4133	6.1231	1.4962
5	17.3283	11.0414	8.8671	2.0142	6.4353	**-0.0001**	1.5651	0.2370
6	19.3872	15.0225	12.4469	0.9936	5.5827	0.9101	**-0.0000**	0.5060
7	24.0769	17.2908	12.5625	3.9848	5.3919	0.3620	1.8303	**-0.0000**

actual failure mode

Figure 2: Global minima of the LMI's for the discrete fault case: A positive entry indicates infeasibility of LMI (6) for the corresponding values of $f_3f_2f_1$. Thus presence of the associated fault can be ruled out.

be explained by the combination (nominal model, uncertainty).

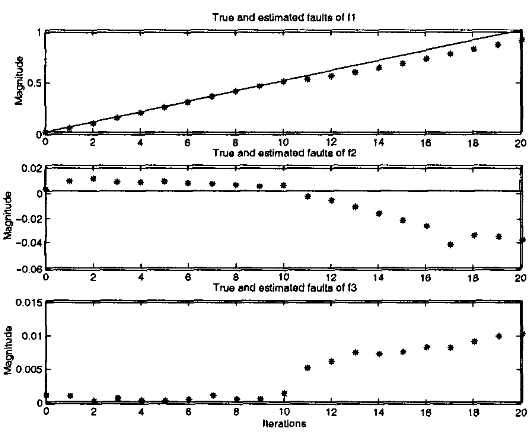

Figure 3: Continuous fault case when $f_1 = a$, $f_2 = 0$, and $f_3 = 0$ (–: true, *:estimated)

Figure 3 shows simulation results for the case $f = [a\ 0\ 0]$. To simulate gradual onset of the fault, the actual value of f_1 was incremented from 0 to 1, in steps of 0.05. Similar results, shown in Figures 4 and 4, were obtained in the cases $f = [a\ a\ 0]$ and $f = [a\ a\ a]$. In all cases, the values of f_i estimated by the proposed method closely match the actual values of the parameters.

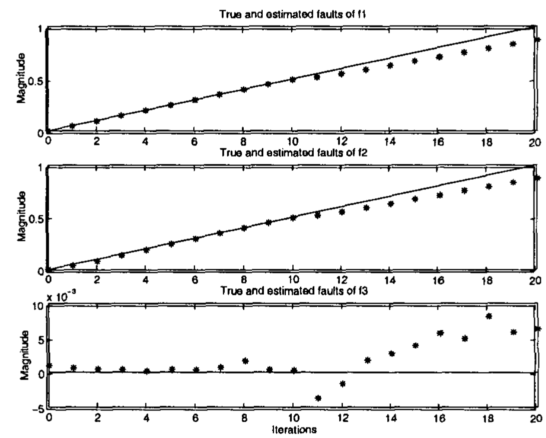

Figure 4: Continuous fault case when $f_1 = a$, $f_2 = a$, and $f_3 = 0$ (–: true, *:estimated)

5 Conclusion

In this paper we considered the problem of robust fault detection and isolation for systems described by a parametrized fault model and subject to multiplicative dynamic uncertainty. Our main result shows that for these systems, fault detection can indeed be accomplished from noisy measurements of the output to a known input, by exploiting Carathéodory-Fejér's theorem to reduce the problem to an LMI feasibility form. Further, in the case of hard faults, this approach allows for identifying the failure mode. These results were illustrated with a practical example. Research is currently under way seeking to extend these results to time–varying systems and to more general uncertainty structures.

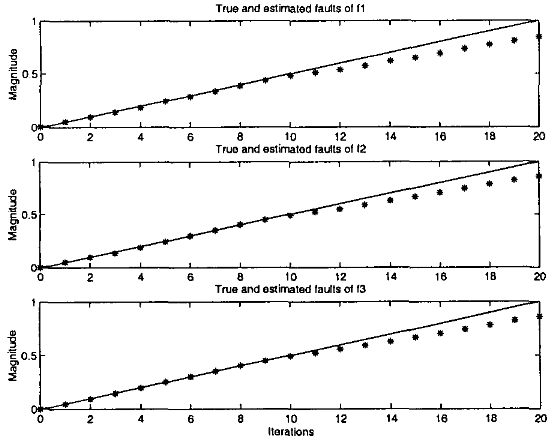

Figure 5: Continuous fault case when $f_1 = a$, $f_2 = a$, and $f_3 = a$ (–: true, *:estimated)

Acknowledgements

This work was supported by NSF grants ECS-9907051 and 0115946, AFOSR grant F49620–00–1–0020, and Korea Research Foundation grant KRF-2001-013-200074.

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