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$\mathcal{H}_2/\mathcal{H}_\infty$ Filtering: Theory and an Aerospace Application

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Abstract

In many filtering problems of practical interest, some of the noise signals satisfy the assumptions of $\mathcal{H}_2$ (Kalman–Bucy) filtering, while others can be more accurately modeled as bounded energy signals (hence more amenable to an $\mathcal{H}_\infty$ filtering approach). These problems may be addressed by considering a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem. In this paper we present a novel theory which solves the mixed problem *exactly* and in a computationally efficient way. The applicability of the theory is illustrated by designing a filter to estimate the states of an aircraft flying through a downburst.

1. Introduction

State estimation of dynamic systems in the presence of process noise and based upon noisy measurements pose an important problem in many engineering applications. A natural way of assessing the performance of a given filter is by considering its effect on the norm of the error signal $\{e_k\}$, and different performance indices may be defined, depending on the a priori assumptions on the noise signal. In the classical $\mathcal{H}_2$ filtering approach (Wiener-Hopf or Kalman-Bucy filtering) the $\mathcal{L}_2$ norm of the error is minimized, under the assumption that the noise characteristics are known [1] (in the sense that the noise is either random with known statistical properties or has a fixed and known spectrum.) Alternatively, during the last few years considerable interest has been given to an $\mathcal{H}_\infty$ filtering approach. This approach does not require a-priori knowledge of the noise statistics; rather the noise is only assumed to have a bounded (possibly weighted) $\mathcal{L}_2$-norm, but it may otherwise be arbitrary.

In practice, noise inputs into a dynamic system may be a combination of the two above mentioned types: noises with known spectral characteristics (often modeled as Markov processes) and noises with unknown spectral characteristic. This motivated the introduction of the $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem [2, 4], in which both noise characteristics are considered simultaneously. Examples in which the mixed filtering problem may be useful can be found in aerospace applications. For instance, during flight, an aircraft often encounters winds with varying types of profiles, including windshear. The latter are attributed to wind profiles with large changes (gradients) in the wind direction and speed. Downburst, one of the most threatening types of windshear, is a mass of cold air that descends to the ground in a column creating windshear and downdrafts.

The purpose of this paper is to present an *exact* solution to the $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem together with an aerospace application, namely, the estimation of the states of an aircraft flying through a downburst. This paper is and abridged version of [7], where both the theory and the example are discussed in detail.

2. Problem formulation

Consider a discrete, linear, time-invariant system with a state space model:

$$\begin{array}{rcll} x_{k+1} & = & Ax_k & + \; Bw_k \\ \check{z}_k & = & C_1x_k & \\ y_k & = & C_2x_k & + \; Dw_k \\ e_k & = & \check{z}_k - \hat{z}_k . & \end{array} \qquad (2-1)$$

were $w \in \mathbb{R}^{m_w}$ is a vector containing both process and measurement noise and $y \in \mathbb{R}^{m_y}$ is the vector of measurements. The filtering problem is to

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produce an estimate $\hat{z}$ of the signal $\check{z} \in \mathbb{R}^{m_z}$ from the measurements y. It will be assumed throughout that the initial state x_0 is known, and without loss of generality, we will take $x_0 = 0$.

A filter F is said to be admissible if it is real rational, stable and, given any initial state $x(0)$, $\lim_{t\to\infty} e(t) = 0$, with $\hat{z} = Fy$. It will be assumed in what follows that the pair (A, C_2) is detectable.

Suppose that (with some abuse of notation) the noise is partitioned as $\left[w^T \;\; v^T\right]^T$, where $w \in \mathcal{L}_2^{m_w}$ is such that $\|w\|_2 < 1$ but otherwise unknown (therefore all weighting functions reflecting any further information are absorbed into the process equations) and the m_v dimensional signal v is assumed to be a zero-mean, Gaussian noise with unit covariance. The state and measurement equations become:

$$\begin{array}{rclcl} x_{k+1} & = & Ax_k & + & B_\infty w_k + B_2 v_k \\ y_k & = & C_2 x_k & + & D_\infty w_k + D_2 v_k. \end{array} \quad (2-2)$$

Let $T_{ew}(z, F)$, $S_{ev}(z, F)$ denote the transfer matrices between the noise signals w and v and the error signal e, resulting from an admissible linear time invariant filter F. Then, the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problems considered in this paper are the following:

Problem 1 *(Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem.) Compute*

$$\psi^2 \doteq \inf\left\{\|S_{ev}\|_2^2 + \|T_{ew}\|_\infty^2 \;\; s.t. \;\; F \text{ is admissible}\right\} \quad (2-3)$$

and given a number $\epsilon > 0$, find an admissible filter F such that $\|S_{ev}\|_2^2 + \|T_{ew}\|_\infty^2 \leq \psi^2 + \epsilon$.

Problem 2 *(Constrained $\mathcal{H}_2$ filtering problem.) Compute*

$$\mu^o \doteq \inf\left\{\|S_{ev}\|_2 \, s.t. \;\; F \text{ is admissible}, \;\; |T_{ew}\|_\infty \leq \gamma\right\} \quad (2-4)$$

and given a number $\epsilon > 0$, find an admissible filter F such that $|T_{ew}\|_\infty \leq \gamma + \epsilon$ and $\|S_{ev}\|_2 \leq \mu^o + \epsilon$.

At the present time, no analytical solutions are known for these problems. A problem related to 2-4, where $\|S_{ev}\|_2$ is replaced by an upper bound was formulated in [2]. In [4] it was shown that this modified problem can be cast into a convex optimization over a bounded set of matrices and solved using non–differentiable optimization techniques. Unfortunately, there is little information regarding the gap between this upper bound and the true $\mathcal{H}_2$ cost.

For simplicity, we will assume *w.l.o.g.* that $\gamma = 1$. Consider a filter F_2 with a state space realization

$$\begin{array}{rcl} \hat{x}_{k+1} & = & (A - LC_2)\hat{x}_k + Ly \\ \hat{z}_k & = & C_1 \hat{x}_k \end{array} \quad (2-5)$$

where the observer gain L is such that $A - LC_2$ is stable. Then, from Theorem 2.1 in [4], F is an admissible filter if and only if $F = F_2 + \hat{Q}\hat{T}_2$, where

$$\hat{T}_2 = \left(\begin{array}{c|c} A - LC_2 & L \\ \hline -C_2 & I \end{array}\right). \quad (2-6)$$

and $\hat{Q}$ is an asymptotically stable transfer matrix. The transfer function $\left[\begin{array}{cc} T_{ew} & S_{ev} \end{array}\right]$ from $\left[\begin{array}{c} w \\ v \end{array}\right]$ to e therefore results from an admissible filter if and only if

$$T_{ew} = T_1 - QT_2, \;\; S_{ev} = V_1 - QV_2. \quad (2-7)$$

Let $A_L \doteq A - LC_2$, $B_L = B_\infty - LD_\infty$.

Let X be the stabilizing solution of the algebraic Riccati equation

$$X = AXA^T + B_\infty B_\infty^T - (AXC_2^T + B_\infty D_\infty^T)(D_\infty D_\infty^T + C_2 X C_2^T)^{-1}(C_2 X A^T + D_\infty B_\infty^T) \quad (2\text{-}8)$$

To facilitate exposition, it is assumed in what follows that X is non-singular, a condition that can be enforced by elliminating the states associated with the null space of X [7]. Taking $L = (AXC_2^T + B_\infty D_\infty^T)(D_\infty D_\infty^T + C_2 X C_2^T)^{-1}$ and $R_c = D_\infty D_\infty^T + C_2 X C_2^T$ makes T_2 into a co-inner transfer matrix. If T_2 is not square, let $C_\perp$, $D_\perp$ solve the equations $C_\perp X A^T + D_\perp B_\infty^T = 0$, $C_\perp X C_2^T + D_\perp D_\infty^T = 0$ and $C_\perp X C_\perp^T + D_\perp D_\perp^T = I$. Then,

$$\|T_1 - QT_2\|_\infty = \left\| \begin{array}{c} G_1 + Q^\sim \\ G_2 \end{array} \right\|_\infty \quad (2-9)$$

where

$$\left[\begin{array}{c} G_1 \\ G_2 \end{array}\right] = \left(\begin{array}{c|c} A_L & A_L X C_1^T \\ \hline R_c^{-1/2} C_2 & R_c^{-1/2} C_2 X C_1^T \\ C_\perp & C_\perp X C_1^T \end{array}\right). \quad (2-10)$$

3. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Filtering

By using 2-7, the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem 2-4 can be precisely stated as solving the following convex optimization problem:

$$\mu^o = \inf_{Q \in \mathcal{RH}_\infty} \|S_{ev}\|_2 = \inf_{Q \in \mathcal{RH}_\infty} \left(\sum_{i=0}^{\infty} \|S_i\|_F^2 \right)^{\frac{1}{2}} \qquad (3-1)$$

subject to:

$$\|T_1 - Q(z)T_2\|_\infty \leq 1 \qquad (3-2)$$

where $\|.\|_F$ denotes the Frobenious norm and where $\{S_i\}$ are the coefficients of the impulse response of S_{ev} (recall that S_{ev} is stable).

Remark: 1 *In the sequel we will assume that* $\inf_{Q \in \mathcal{H}_\infty} \|T_1 - QT_2\|_\infty \doteq \gamma^* < 1$. *This simplifies some technical aspects of the problem.*

Note that problem 3-1 is an *infinite-dimensional* optimization problem. We will show that the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem can be solved by considering a sequence of problems, each one requiring the solution of a finite dimensional convex optimization problem.

Problem 3 *($\mathcal{H}_2/\mathcal{H}_{\infty,\delta}$ Filtering Problem.) Given $V_1(z), V_2(z), T_1(z), T_2(z) \in \mathcal{RH}_{\infty,\delta}$, find*

$$\mu_\delta^o = \inf_{Q \in \mathcal{H}_{\infty,\delta}} \|V_1 - QV_2\|_2, \ s.t. \left\| \begin{matrix} G_1 + Q^\sim \\ G_2 \end{matrix} \right\|_{\infty,\delta} \leq 1 \qquad (3-3)$$

where $\delta < 1$ and $G \in \mathcal{RH}_{\infty,\delta}$ is as in 2-10.

Remark: 2 *From the maximum modulus theorem, any solution Q to $\mathcal{H}_2/\mathcal{H}_{\infty,\delta}$ is an admissible solution for $\mathcal{H}_2/\mathcal{H}_\infty$, implying $\mu_\delta^o \geq \mu^o$.*

Remark: 3 *Problem $\mathcal{H}_2/\mathcal{H}_{\infty,\delta}$ can be thought as solving problem $\mathcal{H}_2/\mathcal{H}_\infty$ with the additional constraint that all the poles of the error system must be inside the disk of radius δ. A parametrization of all admissible filters such that T_i, V_i satisfy this additional constraint can be obtained from 2-7 by simply changing the stability region from the unit-disk to the δ-disk using the transformation $z = \delta\tilde{z}$ before performing the factorization. Furthermore, by combining this transformation with the co-inner factorization, the resulting $T_2(z)$ satisfies $T_2(\delta z)T_2(\frac{1}{\delta z})^\sim = I$.*

Lemma 1 *Consider an increasing sequence $\delta_i \to 1$. Let μ^o and μ_i denote the solution to problems $\mathcal{H}_2/\mathcal{H}_\infty$ and $\mathcal{H}_2/\mathcal{H}_{\infty,\delta_i}$ respectively. Then the sequence $\mu_i \to \mu^o$.*

Lemma 2 *For every $\epsilon > 0$, there exists $N(\epsilon, \delta)$ such that if $Q \in \mathcal{H}_{\infty,\delta}$ satisfies the constraint $\left\| G(z) + \begin{bmatrix} Q(z)^\sim \\ 0 \end{bmatrix} \right\|_{\infty,\delta} \leq 1$, it also satisfies $\sum_{i=N}^{\infty} \|S_i\|_F^2 \leq \epsilon^2$, where S_k denote the coefficients of the impulse response of $S_{ev} = V_1 - QV_2$.*

Given $S \in \mathcal{H}_\infty$, $S(z) = \sum_{i=0}^{\infty} S_i z^{-i}$, consider the projection $\mathcal{P}_N(S(z)) \doteq \sum_{i=0}^{N-1} S_i z^{-i}$. Since $\mathcal{P}_N(S_{ev}) = \mathcal{P}_N(V_1) - \mathcal{P}_N(V_2)\mathcal{P}_N(Q)$, the first n matrices S_i may be written as a *finite* affine combination of $Q_0, \cdots, Q_{n-1}$.

Lemma 3 *Let N be as in Lemma 2, and consider the following optimization problem:*

$$\min_{Q \in \mathcal{RH}_{\infty,\delta}} \|\mathcal{P}_N(V_1 - QV_2)\|_F \quad \text{subject to} \quad \left\| G(z) + \begin{bmatrix} Q(z)^\sim \\ 0 \end{bmatrix} \right\|_{\infty,\delta} \leq 1 \qquad (3-4)$$

Let Q^* denote the optimal solution and define $\mu_\delta^\epsilon = \|S_{ev}^*\|_2$. Then $\mu_\delta^o \leq \mu_\delta^\epsilon \leq \mu_\delta^o + \epsilon$.

By combining the results of Lemmas 1, 2 and 3, the following result is now apparent:

Lemma 4 *Consider an increasing sequence $\delta_i \to 1$. Let μ^o and $\mu_{\delta_i}^\epsilon$ denote the solution to problems $\mathcal{H}_2/\mathcal{H}_\infty$ and $\mathcal{H}_2/\mathcal{H}_{\infty,\delta_i}^\epsilon$ respectively. Then the sequence $\mu_{\delta_i}^\epsilon$ has an accumulation point $\hat{\mu}_\epsilon$ such that $\mu^o \leq \hat{\mu}_\epsilon \leq \mu^o + \epsilon$.*

For the proofs of **Lemmas** 1–4, see [7].

3.1. The $\mathcal{H}_\infty$ Constraint

In this section we show that each truncated problem considered in the previous section can be *exactly* solved by solving a finite dimensional convex optimization problem and an unconstrained $\mathcal{H}_\infty$ filtering problem. To establish this result we will derive first a necessary and sufficient condition for the feasibility of 3-2 when the first N parameters in the expansion $Q(z) = Q_0 + Q_1 z^{-1} + \cdots + Q_{n-1} z^{-(n-1)} + \cdots$ are fixed. An analogous result was obtained in [8] for the SISO one-block control problem, and subsequently extended in [6] to the one-block MIMO and four-block control problems. Although the problem relevant for $\mathcal{H}_\infty$ filtering is a special case of the latter, an alternative simpler derivation was given in [7]. Assume that $\|G_2\|_\infty < 1$. Define $M = I - C_\perp X C_1^T C_1 X C_\perp^T$. Then the algebraic Riccati equation

$$\begin{aligned} L_c &= A_L L_c A_L^T + A_L X C_1^T C_1 X A_L^T \\ &+ (A_L L_c C_\perp^T + A_L X C_1^T C_1 X C_\perp^T) \\ \times (M &- C_\perp L_c C_\perp)^{-1} (C_\perp L_c A_L^T + C_\perp X C_1^T C_1 X A_L^T) \end{aligned} \quad (3\text{-}5)$$

has a stabilizing solution $L_c \geq 0$.

Theorem 1 *There exists a $Q_{tail} \in \mathcal{RH}_\infty$, such that 3-2 holds for $Q(z) = Q_0 + Q_1 z^{-1} + \cdots + Q_{n-1} z^{-(n-1)} + z^{-n} Q_{tail}$ if and only if $\overline{\sigma}(W_1) \leq 1$, with*
$W_1 \doteq$

$$\begin{bmatrix} l_o A_L^n l_c & l_o H_n & l_o H_{(n-1)} & \cdots & l_o H_2 & l_o H_1 \\ C_\perp A_L^{n-1} l_c & C_\perp H_{n-1} & C_\perp H_{n-2} & \cdots & C_\perp H_1 & C_\perp H_0 \\ C_\perp A_L^{n-2} l_c & C_\perp H_{n-2} & H_{n-3} & \cdots & C_\perp H_0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_\perp^T l_c & C_\perp H_0 & 0 & \cdots & 0 & 0 \\ \hat{C}_2 A_L^{n-1} l_c & \hat{C}_2 H_{n-1} & \hat{C}_2 H_{n-2} & \cdots & \hat{C}_2 H_1 & \hat{C}_2 H_0 + Q_0 \\ \hat{C}_2 A_L^{n-2} l_c & \hat{C}_2 H_{n-2} & \hat{C}_2 H_{n-3} & \cdots & \hat{C}_2 H_0 + Q_0^T & Q_1^T \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{C}_2 l_c & \hat{C}_2 H_0 + Q_0^T & Q_1^T & \cdots & Q_{n-2}^T & Q_{n-1}^T \end{bmatrix}$$

where $H_i \doteq A_L^i X C_1^T$, $\hat{C}_2 \doteq R_c^{-1/2} C_2$ and l_c, l_o are the positive square roots of L_c and L_o respectively.

4. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Filter Design

Combining Lemma 4 and Theorem 1 yields the main result of the paper:

Theorem 2 *A suboptimal solution to the mixed $\mathcal{H}_2/\mathcal{H}_{\infty,\delta}$ filtering problem 3-3, with cost $\mu_\delta \leq \mu_\delta^\epsilon \leq \mu_\delta + \epsilon$ is given by $\hat{Q} = \hat{Q}^N + z^{-N} \hat{Q}_{tail}^N$ where $\hat{Q}^N(z) = \sum_{i=0}^{N-1} \hat{Q}_i z^{-i}$ solves the finite dimensional convex optimization problem:*

$$\hat{Q}^N(z) = \underset{\|W_1\|_2 \leq 1}{\operatorname{argmin}} \|\mathcal{P}_N(V_1 - QV_2)\|_F \qquad (4-1)$$

and Q_R solves the approximation problem

$$\min_{Q_R \in \mathcal{RH}_{\infty,\delta}} \|T_1(z) + Q_F^o T_2(z) + z^{-N} Q_R(z) T_2(z)\|_{\infty,\delta} \qquad (4-2)$$

where $N(\epsilon, \delta)$ is selected according to Lemma 2.

From Theorem 2 it follows that a suboptimal solution to the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem 2-4 (with cost arbitrarily close to the optimum) can be found using the following iterative algorithm:

1. *Data:* An increasing sequence $\delta_i \to 1, \epsilon > 0, \nu > 0$.
2. Solve the unconstrained $\mathcal{H}_2$ filtering problem (using the standard Kalman filtering theory). Compute $\|T_{ew}\|_\infty$. If $\|T_{ew}\|_\infty \leq \gamma$ stop, else set $i = 1$.
3. For each i, find a suboptimal solution to problem 3-3 proceeding as follows:
   (a) Obtain $T_i(z), V_i(z) \in \mathcal{RH}_{\infty,\delta_i}$, with $T_2(z)$ co–inner in $\mathcal{RH}_{\infty,\delta_i}$. This can be accomplished by using the change of variable $z = \delta_i \tilde{z}$ before performing the factorization 2-7.
   (b) Compute $N(\epsilon, \delta_i)$ from Lemma 2.
   (c) Find $Q(z)$ using Theorem 2.
4. Compute $\|T_{ew}(z)\|_\infty$. If $\|T_{ew}(z)\|_\infty \geq \gamma - \nu$ stop, else set $i = i + 1$ and go to 3.

Remark: 4 *At each stage the algorithm produces a feasible solution to problem 2-4, with cost μ_i which is an upper bound of the optimal cost μ^o.*

4.1. Special Case: The One–Block Problem

Although in general problem 2-3 must be solved in an iterative fashion, by solving a sequence of problems of the form 2-4, in the special case where the number of measurements equals the number of noise signals with bounded spectra (i.e. $m_y = m_w$) can be solved by exploiting the results of Lemma 4 and Theorem 1. Due to space limitations, an explicit result is omitted. The interested reader is refered to [7] for a detailed exposition.

5. An Aerospace Application

In this section we illustrate the theory by designing a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filter to estimate the states of a generic four-engined executive jet plane flying through a moderate near to the ground downburst after take–off. The aircraft was initially trimmed for level flight at Mach 0.2 (67.7m/sec) at an altitude of 450m, when the downburst was encountered. A linear model, describing the perturbations of flight variables relative to the initial trim conditions in the vertical plane, i.e., the longitudinal model, of a generic four-engined executive jet was used [5]. The downburst was modeled by two rings of the simplified ring-vortex downburst model [3].

5.1. The Designs

The wind profiles of the downburst show that the energy of the horizontal and vertical wind components is concentrated in the low frequency range. These signals constitute the process noise inputs for the simulation; their bandwidth depends on the speed of the aircraft and the size of the downburst. To model this wind, a low-pass weighting function was added to each of the input channels, with cutoff frequency calculated for the expected speed range and an estimation of the expected size of the downburst (approximately 0.2Hz in our example.)

The measurement noise was assumed to be a zero-mean Gaussian sequence, with covariance equal to $diag\{1,2\}^o/sec$. For comparison, an $\mathcal{H}_\infty$ filter was first designed, by minmizing $\|T_{ew} \quad \kappa S_{ev}\|_\infty$. Taking $\kappa = 1$ gave $\|S_{ev}\|_2 = .107$ but $\|T_{ew}\|_\infty = 54.8$. Taking $\kappa = .001$ gave $\|T_{ew}\|_\infty = .90$ but $\|S_{ev}\|_2 = 313.7$. Further reduction of κ produced a marginal reduction on $\|T_{ew}\|_\infty$ and a substantial increase on $\|S_{ev}\|_2$. A seemingly suitable tradeoff was achieved for $\kappa = .005$, giving $\|S_{ev}\|_2 = 186.0$ and $\|T_{ew}\|_\infty = 1.16$.

For the $\mathcal{H}_2/\mathcal{H}_\infty$ design, the objective was to minimize $\|S_{ev}\|_2$ subject to $\|T_{ew}\|_\infty \leq 1.1$. For this problem, the parameterization described in Section 2 is ill-posed, and therefore the constraint was replaced by $\|T_{ev} \quad \kappa S_{ew}\|_\infty \leq 1.1$, with $\kappa = .001$. The final mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filter was designed for $n = 120$, giving $\|S_{ev}\|_2 = 27.82$, six times smaller than the one obtained by the $\mathcal{H}_\infty$ design. The optimal $Q(z)$ had 240 terms, but a reduced order model with 7 states was computed, which achieves a 2 norm of 28.61. The corresponding filter has order 15. Reducing this filter to order 8 further increased the 2 norm to 28.73, about 3% more than the optimal solution. Fig. 5.1 shows the performance of the filter with 15 states.

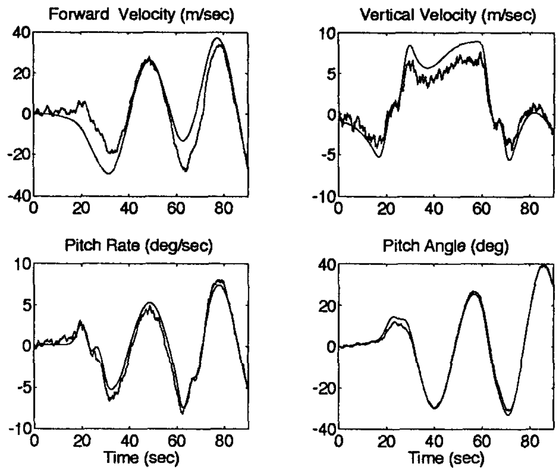

Figure 1: Performance of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filter

6. Conclusions

In this paper we have studied the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problem, and illustrated its applicability with an aerospace example. First we formulated the mixed problem and the closely related problem of minimizing an $\mathcal{H}_2$ norm subject to an $\mathcal{H}_\infty$ norm constraint. Then we showed that the solution to a modified version of the latter problem can be approximated arbitrarily close by the solution to a finite dimensional convex optimization. Finally we proved that, in the limit, the solution to the modified problem converges to the solution of the original one, in the sense that the same level of performance is achieved. The approach was then applied to the estimation of the states of an aircraft flying through a downburst.

References

[1] B. D. O. Anderson and J. Moore. *Optimal Filtering.* Prentice Hall, 1979.

[2] D. Bernstein and W. Haddad. Steady state kalman filtering with an $\mathcal{H}_\infty$ error bound. *Systems and Control Letters*, 12:9–16, 1989.

[3] M. Ivan. A Ring-Vortex Downburst Model for Flight Simulations. *AIAA Journal of Aircraft*, 23(3):232–236, 1986.

[4] P. Khargonekar and M. Rotea. Mixed $\mathcal{H}_\infty/\mathcal{H}_2$ Filtering. In *Proceedings of the 31st Conference on Decision and Control*, pages 2299–2304, Tucson, AZ, Dec. 1992. IEEE.

[5] D. McLean. *Automatic Flight Control Systems.* Prentice Hall, 1990.

[6] H. Rotstein and A. Sideris. $\mathcal{H}_\infty$ optimization with time domain constraints. *IEEE Transactions on Automatic Control*, 1993. To be Published.

[7] H. Rotstein, M. Sznaier, and M. Idan. $\mathcal{H}_\infty/\mathcal{H}_2$ filtering: Theory and an aerospace application. *International Journal of Robust and Nonlinear Control*, 1993. Submitted for publication.

[8] A. Sideris and H. Rotstein. Single input-single output $\mathcal{H}_\infty$-control with time domain constraints. *Automatica*, 29(4):969–983, 1993.