A Risk–Adjusted Approach to Model (In)validation

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Abstract

This paper presents a risk–adjusted approach to the problem of model (in)validation of LTI systems subject to structured dynamic uncertainty entering the model in LFT form. The proposed method proceeds by sampling the set of admissible uncertainties, with the aim of finding at least one element that together with the candidate model can reproduce the experimental data. If so, the model is not invalidated by the experimental evidence. Otherwise, if no such element exists, the model is invalidated by the data with a certain probability. As we show in the paper, given $\varepsilon > 0$, it is possible to determine *a priori* the number of samples so that the probability of invalidating a valid model is below ε. Thus, by introducing a relaxation in terms of this risk ε, we can overcome the computational complexity associated with model invalidation in the presence of structured uncertainties.

1 Introduction

This paper presents a risk–adjusted approach for time–domain based model (in)validation of Linear Time Invariant (LTI) systems subject to structured dynamic uncertainty, entering the model as a Linear Fractional Transformation (LFT). Formally, this problem can be stated as follows: Given experimental data corrupted by noise, find whether or not this data could have been produced by a combination of the candidate model and some uncertainty in a given uncertainty set. If the answer is negative, then the model is said to be invalidated by the data and is rejected; otherwise is said to be not invalidated by the experimental evidence available so far.

Model (in)validation of LTI systems has been extensively studied in the past decade (see for instance [2, 4, 5, 8] and references therein). The main result shows that in the case of unstructured uncertainty and LFT dependence, model (in)validation reduces to a convex optimization problem that can be efficiently solved. In the case of structured uncertainty, the problem leads to bilinear matrix inequalities, and has been shown to be NP–hard in the number of uncertainty blocks in [7]. However, (weaker) necessary conditions in the form of LMIs are available, by reducing the problem to the (in)validation of a scaled model subject to a scaled unstructured uncertainty ([2, 7]).

Motivated by earlier results for sampling LTI causal bounded operators in $\mathcal{BH}_\infty$ ([3]), in this paper we seek to overcome the computational complexity of the problem by pursuing a *risk–adjusted* approach. The proposed technique proceeds by uniformly sampling the set of admissible uncertainties, with the aim of finding at least one that together with the candidate model can reproduce the experimental data. If no such uncertainty can be found, then we can conclude that, with a certain probability, the model is invalid. As shown in the sequel, given any $\varepsilon > 0$, we can compute *a priori* the number of samples so that the probability of rejecting a valid model is below ε. Thus, by introducing a (small) risk of rejecting a possibly *good* candidate model, we can substantially alleviate the computational complexity entailed in validating models subject to structured uncertainty.

The paper is organized as follows. Section 2 introduces the notation used through the paper as well as some required results, and states the model (in)validation problem. Section 3 presents the proposed method. Section 4 illustrates the results of the paper with a simulated example. Finally, Section 5 contains some concluding remarks and directions for future research.

2 Preliminaries

2.1 Notation

In the sequel, $\mathbf{R}$ represents the set of real numbers, $\mathbf{x} \in \mathbf{R}^m$ denotes a real–valued column vector, $\mathbf{x}^T$ a real–valued row vector, $\mathbf{x}(k)$ its k-*th* element and $\|\mathbf{x}\|_p$ its p-norm:

$$\|\mathbf{x}\|_p \doteq \left(\sum_{k=1}^{m} |\mathbf{x}(k)|^p\right)^{\frac{1}{p}} \quad p \in [1,\infty)$$
$$\|\mathbf{x}\|_\infty \doteq \max_{k=1,\ldots,m} |\mathbf{x}(k)|.$$

Given one–sided, finite vector sequences $x = \{\mathbf{x}_i\}_{i=0}^{N}$ with $\mathbf{x}_i \in \mathbf{R}^m$, $\ell_p^m[0,N]$ denotes the space of bounded sequences in the $\ell_p[0,N]$-norm:

$$\|x\|_{p[0,N]} \doteq \left(\sum_{i=0}^{N} \|\mathbf{x}_i\|_p^p\right)^{\frac{1}{p}} \quad p \in [1,\infty)$$
$$\|x\|_{\infty[0,N]} \doteq \max_{i=0,\ldots,N} \|\mathbf{x}_i\|_\infty,$$

and ℓ_p^m denotes the extended Banach space of infinite sequences bounded in the ℓ_p-norm:

$$\|x\|_p \doteq \left(\sum_{i=0}^{\infty} \|\mathbf{x}_i\|_p^p\right)^{\frac{1}{p}} \quad p \in [1,\infty)$$
$$\|x\|_\infty \doteq \sup_i \|\mathbf{x}_i\|_\infty.$$

As usual, $\ell_p[0,N]$ and ℓ_p stand for the case of scalar sequences.

$\mathcal{L}_\infty$ denotes the Lebesgue space of complex-valued matrix functions essentially bounded on the unit circle $|z|=1$, equipped with the norm:

$$\|G\|_\infty \doteq ess \sup_{|z|=1} \overline{\sigma}(\mathbf{G}(z))$$

Similarly $\mathcal{H}_\infty$ denotes the subspace of functions in $\mathcal{L}_\infty$ with a bounded analytic continuation inside the unit disk $|z|<1$, with norm:

$$\|G\|_\infty \doteq ess \sup_{|z|<1} \overline{\sigma}(\mathbf{G}(z)).$$

Also of interest is the Banach space $\mathcal{H}_{\infty,\rho}$ of functions in $\mathcal{H}_\infty$ which have analytic continuation inside the disk of radius $\rho > 1$, with norm:

$$\|G\|_{\infty,\rho} \doteq \sup_{|z|<\rho} \overline{\sigma}(\mathbf{G}(z)).$$

Given a normed space $\{\mathcal{X}, \|\cdot\|_{\mathcal{X}}\}$, $\mathcal{BX}(\gamma)$ denotes the open γ-ball in $\mathcal{X}$:

$$\mathcal{BX}(\gamma) = \{x \in \mathcal{X} : \|x\|_{\mathcal{X}} < \gamma\}$$

and $\overline{\mathcal{BX}}(\gamma)$ its closure. In the sequel, $\mathcal{H}_\infty$ and $\|\cdot\|_\infty$ stand for the case $\rho = 1$, and $\mathcal{BX}$ for the unit ball in $\mathcal{X}$.

This paper considers finite-dimensional, causal, discrete-time, LTI systems bounded in ℓ_2^m or exponentially stable, i.e.

$$\|S\|_{\ell_2^m \to \ell_2^m} \doteq \sup_{\|u\|_{\ell_2^m} \neq 0} \frac{\|S * u\|_{\ell_2^m}}{\|u\|_{\ell_2^m}} < \infty,$$

where $*$ stands for convolution. Any such system will be represented by its convolution kernel $\{\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2, \cdots\}$, i.e.:

$$\mathbf{y}_k = \sum_{j=0}^{k} \mathbf{S}_{k-j}\mathbf{u}_j,$$

or by its $\mathcal{Z}$-transform evaluated at $1/z$:

$$S(z) = \sum_{k=0}^{\infty} \mathbf{S}_k z^k,$$

where $S(z) \in \mathcal{H}_{\infty,\rho}$, for some $\rho > 1$. It is a well known fact that in this case $\|S\|_{\ell_2 \to \ell_2} = \sup_{|z|=1} \overline{\sigma}(S(z))$.

Finally, for a real-valued matrix $\mathbf{A} \in \mathbf{R}^{m \times n}$, $\mathbf{A}^T$ denotes its transpose and $\mathbf{A}^{\frac{1}{2}}$ its square root.

2.2 Some Required Results

The following algorithm, developed by [3], generates N_s uniformly distributed samples over the set $\mathcal{C}$, consisting of all finite impulse responses $h = \{\mathbf{H}_0, \mathbf{H}_1, \ldots, \mathbf{H}_N\}$ so that the function $H(z) \doteq \sum_{k=0}^{N} \mathbf{H}_k z^k$ *can be completed* to belong to $\mathcal{BH}_\infty$. It will used to sample the uncertainty set $\mathbf{\Delta}_{st}(\gamma)$.

Algorithm 1

Let $k=0$. Generate N_s samples uniformly distributed over the set

$$\{\mathbf{H}_0 : \overline{\sigma}(\mathbf{H}_0) \leq 1\}.$$

1. *Let $k := k+1$. For every generated sample $(\mathbf{H}_0^i, \mathbf{H}_1^i, \ldots, \mathbf{H}_{k-1}^i)$, consider the partition*

$$\begin{bmatrix} \mathbf{H}_k^i & \cdots & \mathbf{H}_1^i & \mathbf{H}_0^i \\ \mathbf{H}_{k-1}^i & \cdots & \mathbf{H}_0^i & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{H}_0^i & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_k^i & \mathbf{B} \\ \mathbf{C} & \mathbf{A} \end{bmatrix}$$

   and let the matrices $\mathbf{Y}$ *and* $\mathbf{Z}$ *be a solution for the linear equations*

$$\mathbf{B} = \mathbf{Y}(\mathbf{I} - \mathbf{A}^T\mathbf{A})^{\frac{1}{2}};$$
$$\mathbf{C} = (\mathbf{I} - \mathbf{A}\mathbf{A}^T)^{\frac{1}{2}}\mathbf{Z},$$
$$\overline{\sigma}(\mathbf{Y}) \leq 1, \overline{\sigma}(\mathbf{Z}) \leq 1$$

   with $\mathbf{I}$ *the identity matrix. Let* $\mathbf{J}(\mathbf{H}_0^i, \mathbf{H}_1^i, \ldots, \mathbf{H}_{k-1}^i)$ *be the Jacobian matrix of the linear transformation that maps the set* $\{\mathbf{W} : \overline{\sigma}(\mathbf{W}) \leq 1\}$ *to the set* $\{\mathbf{H}_k : h \in \mathcal{C}\}$. *Generate*

$$\lfloor N_s \mathbf{J}(\mathbf{H}_0^i, \mathbf{H}_1^i, \ldots, \mathbf{H}_{k-1}^i) \rfloor,$$

   samples uniformly over the set $\{\mathbf{W} : \overline{\sigma}(\mathbf{W}) \leq 1\}$, *where* $\lfloor x \rfloor$ *denotes the largest integer smaller than or equal to x. For each of those samples* $\mathbf{W}^i$, *take*

$$\mathbf{H}_k^i = -\mathbf{Y}\mathbf{A}^T\mathbf{Z} + (\mathbf{I} - \mathbf{Y}\mathbf{Y}^T)^{\frac{1}{2}}\mathbf{W}^i(\mathbf{I} - \mathbf{Z}^T\mathbf{Z})^{\frac{1}{2}}.$$

2. *If $k < N$ go to step 1. Otherwise, stop.*

2.3 Problem Statement

Consider Figure 1, which depicts the model (in)validation set-up, as a lower LFT interconnection $\mathcal{F}_l(M, \Delta)$ between model M:

$$M \doteq \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$$

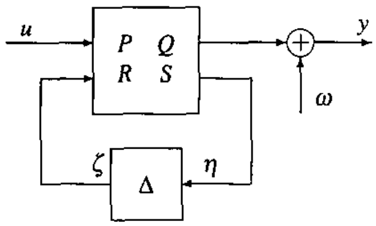

Figure 1: The model (in)validation set–up

and an uncertainty block Δ in some set $\mathbf{\Delta}_{st}$, i.e.

$$
\begin{aligned}
y &= P * u + Q * \zeta + \omega \\
\eta &= R * u + S * \zeta \\
\zeta &= \Delta * \eta
\end{aligned}
$$

or in closed–loop form $\mathcal{F}_l(M,\Delta) = P + Q\Delta(I - S\Delta)^{-1}R$, where I is the identity system of appropriate dimensions.

The block M consists of a nominal model P of the actual system and some description of how uncertainty affects the model (e.g. additive or multiplicative dynamic uncertainty), represented by the blocks Q, R and S. The block Δ represents structured diagonal dynamic uncertainty. Finally, the signals u and y represent an arbitrary but known test input and its corresponding output respectively, corrupted by measurement noise ω in a given set $\mathcal{N}$.

This paper considers the following admissible sets of uncertainties and noise:

$$
\begin{aligned}
\mathbf{\Delta}_{st}(\gamma) &\doteq \{\Delta \colon \Delta = \operatorname{diag}(\Delta_1, \dots, \Delta_l), \\
&\qquad \Delta_i \in \overline{\mathcal{B}}\mathcal{H}_\infty(\gamma), \forall i = 1, \dots, l\} \\
\mathcal{N} &\doteq \mathcal{B}\,\ell_p^m[0,N](\varepsilon_t).
\end{aligned} \tag{1}
$$

Finally, block S is assumed to be bounded by $\|S\|_\infty < \gamma^{-1}$, so that the interconnection $\mathcal{F}_l(M,\Delta)$ is well–posed.

In keeping with the model (in)validation spirit, the goal is to determine whether or not the measured values of the input u and the output y are consistent with the assumed model M and the given set descriptions for the noise $\omega \in \mathcal{N}$ and uncertainty $\Delta \in \mathbf{\Delta}_{st}$. Using these definitions, the model (in)validation problem can be precisely stated as:

Problem 1 *Given the time-domain experiments:*

$$
\begin{aligned}
u &\doteq \{\mathbf{u}_0, \mathbf{u}_1, \dots, \mathbf{u}_N\} \\
y &\doteq \{\mathbf{y}_0, \mathbf{y}_1, \dots, \mathbf{y}_N\}
\end{aligned}
$$

the model M and the a priori *sets $(\mathcal{N}, \mathbf{\Delta}_{st})$ determine whether or not the assumed model together with the* a priori *assumptions could have generated the given experimental data, i.e. whether the consistency set*

$$
\begin{aligned}
\mathcal{T}(y) = \{(\Delta, \dot\omega) \colon \Delta \in \mathbf{\Delta}_{st},\ \omega \in \mathcal{N} \ \ \textit{and} \\
\mathbf{y}_k = \left(\mathcal{F}_l(M,\Delta) * u + \omega\right)_k, k = 0, \dots, N\}
\end{aligned}
$$

is nonempty.

3 Main Results

As mentioned before, in this paper we will pursue a *risk–adjusted* approach, where we (approximately) solve Problem 1 by uniformly sampling the uncertainty set $\mathbf{\Delta}_{st}$ in an attempt to find an element that, together with an admissible noise, explains the observed experimental data. A potential problem here is that the set $\mathbf{\Delta}_{st}$ is infinite dimensional. Note however that given a finite set of $N+1$ input/output measurements, since Δ is causal, *only the first $N+1$ Markov parameters* affect the output y. Thus, rather than having to sample transfer matrices in $\mathbf{\Delta}_{st}$, *we only need to generate samples of the first $N+1$ Markov parameters* of elements of the set $\mathbf{\Delta}_{st}$. This is the key observation that allows to reduce the problem to that of sampling a finite–dimensional set. More precisely, combining this observation with Algorithm 1, leads to the following model (in)validation algorithm:

Algorithm 2 *Given γ_{st}, take N_s samples of $\mathbf{\Delta}_{st}(\gamma_{st})$, $\{\Delta^n(z)\}_{n=1}^{N_s}$, according to the procedure described in Section 2.2.*

1. *At step n, let*

$$
\omega^n \doteq \{(y - \mathcal{F}_l(M,\Delta^n) * u)_k\}_{k=0}^N. \tag{2}
$$

2. *Find whether $\omega^n \in \mathcal{N}$. If so, stop. Otherwise, consider next sample $\Delta^{n+1}(z)$ and go back to step 1.*

Clearly, the existence of at least one $\omega^n \in \mathcal{N}$ is equivalent to $\mathcal{T}(y) \neq \emptyset$. The algorithm finishes, either by finding one admissible uncertainty $\Delta^n(z)$ that makes the model not invalidated by the data or after N_s steps. As shown next, if N_s is *large enough* and $\forall \Delta^n(z)$, $\omega^n \notin \mathcal{N}$, then there is a high probability that the model is invalidated by the available experimental evidence.

Lemma 1 *Let (ε, δ) be two positive constants in $(0,1)$. If*

$$
N_s \geq \frac{\ln(1/\delta)}{\ln(1/(1-\varepsilon))}, \tag{3}
$$

then the probability of rejecting a model which is not invalidated by the data is smaller than ε, and this event occurs with probability greater than $(1-\delta)$.

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Proof: Define the function $f(\Delta^n(z)) \doteq \varepsilon_t - \|\omega^n\|_{p[0,N]}$, with ω^n given by (2). Note that the model is not invalidated by the data whenever one finds at least one $\Delta^n(z)$ so that $f(\Delta^n(z)) > 0$. Equivalently, if $\forall \Delta^n$, $f(\Delta^n(z)) \leq 0$, we might be rejecting a model which is indeed not invalidated by the data. Following [6], if the number of samples is at least of N_s then

$$\text{Prob}\{\text{Prob}\{f(\Delta(z)) > 0\} \leq \varepsilon\} \geq (1-\delta),$$

which yields the desired result. ∎

4 Example

In order to illustrate the proposed method, consider the following *true* system:

$$\hat{G}(z) = \mathcal{F}_l(M, \hat{\Delta}), \tag{4}$$

with block M given by:

$$P(z) = \frac{0.2(z+1)^2}{18.6z^2 - 48.8z + 32.6} \qquad Q(z) = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$R(z) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad S(z) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $\hat{\Delta}(z)$[1]:

$$\begin{bmatrix} \frac{0.125(5.1-4.9z)}{(6.375-3.6250z)} & 0 & 0 \\ 0 & \frac{0.1(5.001-4.9990z)}{(6.15-3.85z)} & 0 \\ 0 & 0 & \frac{0.05(5.15-4.85z)}{(6.95-3.05z)} \end{bmatrix}.$$

Assume we are given $P(z)$ as a candidate model for (4), together with a description of the uncertainty type and how it enters the model in terms of the set $\mathbf{\Delta}_{st}$ and the blocks (Q,R,S) respectively. Our "experimental" data consists of a set of $N = 20$ samples of the impulse response of $\hat{G}(z) = \mathcal{F}_l(P, \hat{\Delta})$, corrupted by additive noise in $\mathcal{N} \doteq \mathcal{B}\,\ell_\infty[0,N](0.0041)$. The noise bound ε_t represents a 10% of the peak value of the true impulse response. The plant, the model and the samples are shown in Figure 2.

The goal is to check whether the given model subject to structured uncertainty is able to reproduce the experimental evidence (u,y), i.e. whether there exist at least one $\Delta \in \mathbf{\Delta}_{st}$ and $\omega = [y_k - (\mathcal{F}_l(M,\Delta) * u)_k]_{k=0}^N \in \mathcal{N}$, by uniformly sampling the set $\mathbf{\Delta}_{st}(\gamma_{st})$. If so, it is also of interest to quantify the minimum size of the uncertainty γ_{st}, so that the model remains not invalidated by the data. If instead no pair (Δ, ω) can be found, then we can reject the model with an *a priori* specified confidence.

Begin by recalling that a lower bound on γ_{st} can be obtained by performing an invalidation test on the assumed model but

[1] Note that $\|\hat{\Delta}\|_\infty = 0.125$.

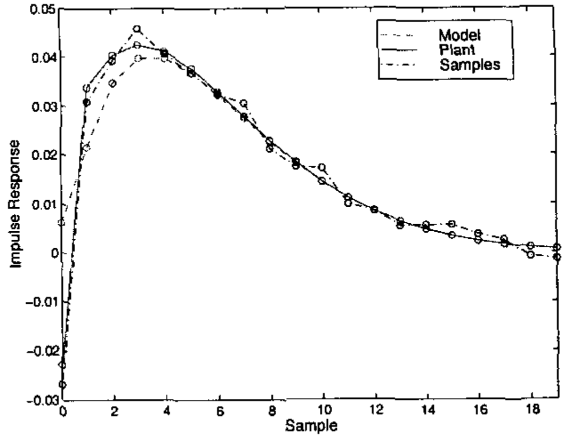

Figure 2: Model, actual plant and samples.

subject to *unstructured* uncertainty $\Delta(s) \in \mathbf{\Delta}_u$ while minimizing over its size, since $\mathbf{\Delta}_u(\gamma) \supset \mathbf{\Delta}_{st}(\gamma)$. As shown in [2], the problem can be recast as a LMI feasibility one and efficiently solved. More precisely, if γ^* is such that $\forall \gamma < \gamma^*$ the model subject to unstructured uncertainty is invalidated by the data, then it will remain so subject to structured uncertainty, and therefore $\gamma^* \leq \gamma_{st}$. This is the rationale behind the method proposed in [2] for model (in)validation with structured uncertainties.

Using the available experimental samples and the *a priori* assumptions, led to a value of γ^* of 0.0158, i.e. $0.0158 \leq \gamma_{st}$. On the other hand, since in this particular example $\hat{\Delta} \in \mathbf{\Delta}_{st}(0.125)$, $\gamma_{st} \leq 0.125$.

To apply the proposed method, we generated 3 sets of $N_s = 6000$ samples over $\mathcal{BH}_\infty$, one for each of the scalar blocks $\Delta_i(z)$, $i = 1,2,3$, which yields one single set of samples $\{\Delta^n(z)\}_{n=1}^{N_s}$ over $\mathbf{\Delta}_{st}$[2]. Following Section 3, at each given value of γ_{st}, we evaluated the function

$$f(\Delta^n) = \varepsilon_t - \|\{(y - \mathcal{F}_l(M,\Delta^n) * u)_k\}_{k=0}^N\|_{\infty[0,N]}$$

for all $\Delta^n \in \mathbf{\Delta}_{st}(\gamma_{st})$.

According to Section 3, $N_s = 6000$ samples guarantee a probability of at least 0.9975 that $\text{Prob}\{f(\Delta) > 0\} \leq 0.001$. Thus, if $\forall \Delta^n$, $f(\Delta^n) < 0$ the model is invalidated by the data with high probability; it is then necessary to increase the value of γ_{st} and continue the (in)validation test. Starting from $\gamma_{st} = 0.0158$, we repeated this test over a grid of 1000 points over the interval $[0.0158, 0.125]$ until we obtained the minimum value of γ_{st} of 0.0775 for which the model is not invalidated by the given experimental evidence.

The proposed approach differs from the one in [2] in that here the invalidation test is performed by searching over $\mathbf{\Delta}_{st}$

[2] The corresponding samples over the set $\mathbf{\Delta}(\gamma_{st})$ were obtained by appropriate scaling of the impulse response of each given sample by γ_{st}.

with the hope of finding one admissible $\Delta \in \mathbf{\Delta}_{st}$ that makes the model not invalid; while there it is done by searching over $\mathbf{\Delta}_u$ and by introducing, at each step, diagonal similarity scaling matrices with the aim of invalidating the model. More precisely, if at step k the model subject to unstructured uncertainty remains *not* invalidated (which is equivalent to the existence of at least one feasible pair $(\zeta, \mathbf{D}_k)$ so that a given matrix $\mathbf{H}(\zeta, \mathbf{D}_k)$ is negative semidefinite, i.e. $\mathbf{H}(\zeta, \mathbf{D}_k) \leq 0$), one possible strategy is to select the scaling $\mathbf{D}_{k+1}$ so as to maximize the trace of $\mathbf{H}$. See [1], Chapter 9, pp. 301–306 for details. However, for this particular example $\mathbf{D}_k = \mathrm{diag}(d_{1k}, d_{2k}, d_{3k})$ and this last condition becomes:

$$\sup_{d_{1k},d_{2k},d_{3k}} -N(1+\frac{1}{\gamma^2})(d_{2k}+d_{3k}) + N(1-\frac{1}{\gamma^2})d_{1k},$$
$$d_{1k} \geq 0,\ d_{2k} \geq 0,\ d_{3k} \geq 0.$$

For $0 < \gamma < 1$, clearly the supremum is achieved at $d_{1k} = 0$, $d_{2k} = 0$ and $d_{3k} = 0$. As an alternative searching strategy, one may attempt to randomly check condition $\mathbf{H}(\zeta, \mathbf{D}_k) \leq 0$ by sampling appropriately the scaling matrices, following [7]. Using 6000 samples, this led to a value of γ_{st} of 0.03105 for which the model is invalidated by the data. For larger values of γ_{st} in $[0.03105, 0.125]$ nothing can be concluded regarding the validity of the model.

On the other hand, our method leads to an interval $[0.0158, 0.0775]$ where, even though we are not completely certain, we can reject the model with a very low risk of actually discarding a valid model[3]. These results suggest that both approaches, rather than competing, can be combined to further reduce these gaps, leading to better informed decisions on whether to reject candidate models.

As a final remark, note that it seems possible to reduce the number of samples required by the proposed method, at the expense of requiring additional *a priori* information on the actual system. This situation may arise for example when it is known that the uncertainty affecting the candidate model is exponentially stable or even real, if the system has uncertain parameters. The former case amounts to sampling $\mathcal{B}\mathcal{H}_{\infty,\rho} \subset \mathcal{B}\mathcal{H}_{\infty}$, $\rho > 1$, while the latter involves samples of constant matrices.

5 Conclusions and Further Research

This paper proposes a method for model (in)validation subject to structured uncertainties, by sampling the set of all admissible uncertainties. The approach is risk-adjusted in the sense that if no pair (Δ, ω) can be found, then the model is invalidated by the data with a certain probability, i.e. we might be rejecting a possibly *good* candidate model. By introducing this relaxation, we overcome the computational complexity associated to the model invalidation problem with structured uncertainties.

Efforts are under way to reduce the computational load of the proposed method, determined mainly by the required number of samples N_s, by adding more information into the problem e.g. in terms of *a priori* assumptions on the admissible set of uncertainties and the probability distribution of the samples $\{\Delta^n\}_{n=1}^{N_s}$.

[3] From a robust control stand-point is always preferrable to discard a valid model than to accept an invalid one.

Acknowledgements

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References

[1] J. Chen and G. Gu. *Control–oriented System Identification : an $\mathcal{H}_\infty$ Approach.* John Wiley & Sons, New York, 2000.

[2] J. Chen and S. Wang. Validation of linear fractional uncertain models: Solutions via matrix inequalities. *IEEE Transactions on Automatic Control*, 41(6):844–849, June 1996.

[3] C. Lagoa, M. Sznaier, and B. R. Barmish. An algorithm for generating transfer functions uniformly distributed over $\mathcal{H}_\infty$ balls. In 40^{th} *IEEE Conference on Decision and Control*, pages 5038–5042, Orlando, Florida, USA, December 2001.

[4] K. Poolla, P. Khargonekar, A. Tikku, J. Krause, and K. Nagpal. A time domain approach to model validation. *IEEE Transactions on Automatic Control*, 39(5): 951–959, May 1994.

[5] R.S. Smith and J.C. Doyle. Model validation: A connection between robust control and identification. *IEEE Transactions on Automatic Control*, 37(7):942–952, July 1992.

[6] R. Tempo, E. W. Bai, and F. Dabbene. Probabilistic robustness analysis: Explicit bounds for the minimum number of samples. In 35^{th} *IEEE Conference on Decision and Control*, pages 3424–3428, Kobe, Japan, December 1996.

[7] O. Toker and J. Chen. Time domain validation of structured uncertainty model sets. In 35^{th} *IEEE Conference on Decision and Control*, pages 255–260, Kobe, Japan, December 1996.

[8] D. Xu, Z. Ren, G. Gu, and J. Chen. LFT uncertain model validation with time and frequency–domain measurements. *IEEE Transactions on Automatic Control*, 44(7):1435–1441, July 1999.