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An LMI Approach to Model (In)Validation of LPV Systems

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Abstract

During the past few years efficient tools have been developed to robustly stabilize Linear Parameter Varying systems. However, a key issue that needs to be addressed in order to apply these techniques to practical problems is the development of techniques that, starting from experimental data, generate and validate suitable models. In this paper we propose a new model validation framework for LPV systems subject to unstructured uncertainty. The main result shows that the problem of establishing consistency between the experimental data and the *a priori* assumptions on the nominal model, the uncertainty description and the error bounds can be recast as an LMI feasibility problem and be efficiently solved. Moreover, the overall computational complexity is similar to that of validating LTI models of comparable size.

1 Introduction and Motivation

During the past few years considerably attention has been devoted to the problem of synthesizing controllers for Linear Parameter Varying Systems, where the state–space matrices of the plant depend on time–varying parameters whose values are not known *a priori* but can be measured by the controller. Assuming that bounds on both the parameter values and their rate of change are known then Affine Matrix Inequalities based conditions are available guaranteeing exponential stability of the system. Moreover, these conditions can be easily used to synthesize stabilizing controllers guaranteeing worst case performance bounds (for instance in an $\mathcal{H}_2$ or $\mathcal{H}_\infty$ sense, see [1, 7, 4, 23, 22] and references therein). These results formalize the intuitively appealing idea of gain scheduling, while avoiding its pitfalls [8, 9, 16, 18, 19].

Clearly, a key issue that needs to be addressed in order to apply these techniques to practical problems is the development of identification methods capable of extracting the appropriate description from experimental data. Control oriented identification of LTI systems is by now relatively mature, and efficient algorithms are available to obtain both models and worst case bounds on the identification error (see for instance [17] and references therein). On the other hand, identification tools for LPV systems are just starting to appear [12, 10, 11, 3, 21]. At this point, these tools address the problem of obtaining a nominal model of the LPV system as well as bounds on the identification error, given a set of measurements corrupted by noise and some *a priori* information on the plant. A complete description, suitable to be used by control synthesis algorithms, combines these models with an appropriate uncertainty description, obtained either from the identification algorithm or from *a priori* information on the plant.

However, before this description can be used by the control engineer, it must be *validated*, based on experimental data. This leads to the following model (in)validation problem: given experimental data, corrupted by additive noise, find whether or not this data could have been produced by the combination of the nominal model and some uncertainty in the uncertainty set. If the answer is negative, then the assumed model does not provide a correct description of the physical system. Model validation of LTI systems has been extensively studied in the past decade (see for instance [20, 15, 6, 24] and reference therein). The main result shows that in the case of unstructured uncertainty entering the plant as an LFT, model validation reduces to a convex optimization problem that can be efficiently solved. In the case of structured uncertainty and LFT dependence the problem leads to bilinear matrix inequalities. However, (weaker) necessary conditions for consistency in the form of LMIs are available [6]. On the other hand, comparable results are not available for the case of LPV systems.

Motivated by our earlier results on control oriented identification of LPV [21] and LTI [14] systems, and by the related work in [24], in this paper we propose a new model validation framework for LPV systems subject to unstructured uncertainty. The main result of the paper shows that the problem of establishing consistency of the experimental data with the *a priori* information (nominal model, uncertainty description and bounds on the measurement error) can be recast as an LMI feasibility problem that can be efficiently solved. Moreover, the overall computational complexity is similar to that of validating LTI models of comparable size.

The paper is organized as follows: in section 2 we introduce the notation and formally state the LPV model validation problem. Section 3 contains the main results. Here we show that the problem can be recast into an LMI feasibility form. These results are illustrated in section 4 with a practical example arising in the context of active vision. Finally, section 5 contains some concluding remarks.

2 Preliminaries

2.1 Notation and preliminary results

By $\mathcal{L}_\infty$ we denote the Lebesgue space of complex valued matrix functions essentially bounded on the unit circle, equipped with the norm

$$\|G(z)\|_\infty \doteq ess \sup_{|z|=1} \overline{\sigma}\left(G(z)\right), \tag{1}$$

where $\overline{\sigma}$ represents the largest singular value. By $\mathcal{H}_\infty$ we denote the subspace of functions in $\mathcal{L}_\infty$ with bounded analytic continuation inside the unit disk and with norm

$$\|G(z)\|_\infty \doteq ess \sup_{|z|<1} \overline{\sigma}\left(G(z)\right). \tag{2}$$

$\mathcal{BH}_\infty$ denotes the unit ball in $\mathcal{H}_\infty$. ℓ_2 denotes the space of square summable sequences $h = \{h_i\}$ equipped with the norm

$$\|h\|_{\ell_2} \doteq \left(\sum_{i=0}^{\infty} h_i^2\right)^{\frac{1}{2}}, \tag{3}$$

and $\ell_\infty(\epsilon)$ denotes the space of bounded sequences equipped with the norm:

$$\|h\|_{\ell_\infty} \doteq \sup_{i\geq 0} |h_i| \leq \epsilon < \infty. \tag{4}$$

Consider now the space $\mathcal{L}(\ell_2)$ of bounded, causal linear time invariant operators in ℓ_2. An element of $\mathcal{L}(\ell_2)$ can be represented by its convolution kernel $\{L_k\}$. The projection operator $\mathcal{P}_n : \mathcal{L}(\ell_2) \to \mathcal{L}(\ell_2)$ is defined by

$$\mathcal{P}_n[L] \doteq \{L_0, L_1, \dots, L_{n-1}, 0, 0, \dots\}. \tag{5}$$

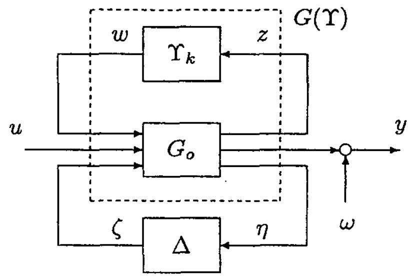

Figure 1: Setup for Model (In)Validation of LPV Systems

Given an operator $L \in \mathcal{L}(\ell_2)$ and its projection $\mathcal{P}_n[L]$, we define its associated (finite) lower Toeplitz matrix as follows:

$$T_L^n = \begin{bmatrix} L_0 & 0 & \dots & 0 \\ L_1 & L_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_{n-1} & L_{n-2} & \dots & L_0 \end{bmatrix}, \tag{6}$$

Similarly, to a given sequence h and its projection in ℓ_2 we associate the matrix:

$$T_h^n = \begin{bmatrix} h_0 & 0 & \dots & 0 \\ h_1 & h_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{n-1} & h_{n-2} & \dots & h_0 \end{bmatrix}. \tag{7}$$

In the sequel, for notational simplicity, the superscript will be omitted when clear from the context.

Finally given a matrix M, M^T denotes its transpose. As usual $M > 0$ ($M \geq 0$) indicates that M is positive definite (positive semi-definite), and $M < 0$ that M is negative definite.

Lemma 1 (Carathéodory-Fejér) *Given a matrix-valued sequence $L_i, i = 0, \dots, n-1$, there exists a causal LTI operator $L(z) \in \mathcal{BH}_\infty$ such that*

$$L(z) = L_o + L_1 z + L_2 z^2 + \dots L_{n-1} z^{n-1} + \dots$$

if and only if $M_c \doteq I - T_L^* T_L \geq 0$

Proof: See for instance [2, 24].

2.2 Model Validation of LPV Systems

Consider the stable discrete time LPV system shown in Figure 1. Here G_o is a known given system, and the signals u and y represent a known test input and the corresponding output corrupted by measurement noise ω. The block $\Upsilon_k = \mathrm{diag}\left(\rho_1 I_{r_1}, \dots, \rho_s I_{r_s}\right)$ represents a set of time–varying parameters, that are unknown *a priori*, but can be measured in real time. Finally, as usual Δ represents bounded dynamic uncertainty. In keeping with the model validation spirit, the goal is to determine whether or not the measured values of the input u, the output y and time–varying parameters Υ_k are consistent with the assumed model G_o and the given set descriptions for the noise w and uncertainty Δ.

In the sequel we consider models $G(\Upsilon)$, uncertainties Δ and noises ω of the form:

$$G(\Upsilon) = \mathcal{F}_u\left[G_o, \Upsilon_k\right] \doteq \begin{bmatrix} P(\Upsilon) & Q(\Upsilon) \\ R(\Upsilon) & S(\Upsilon) \end{bmatrix}$$
$$\Delta \in \mathbf{\Delta} \doteq \{\Delta \in \mathcal{H}_\infty \colon \|\Delta\|_\infty \leq \delta \leq 1\}$$
$$\omega \in \mathcal{N} \doteq \{\omega \in R^N \colon L(\omega) = L_0 + \sum_{k=1}^{n} L_k \omega_{k-1} > 0, \quad L_i > 0, \text{ given}\}, \tag{8}$$

where $\mathcal{F}_{u(\ell)}$ denotes upper (lower) linear fractional transformation (LFT). Moreover, we will assume that $\|S(\Upsilon)\|_{\ell_2 \to \ell_2} < \delta^{-1}$ for all parameter trajectories Υ_k, so that the interconnection $\mathcal{F}_\ell\left[G(\Upsilon), \Delta\right]$ is ℓ_2 stable for all Υ_k. The noise set $\mathcal{N}$ is a generalization of the $\ell_\infty(\epsilon)$ noise set usually considered ([5, 14]), as defined in (4). This more general form allows for taking into consideration correlated noise (see [13] for details).

Using these definitions the LPV model (in)validation problem can be precisely stated as:

Problem 1 *Given the time-domain experiments* $\mathbf{u} = (u_1, \ldots, u_n)$, $\mathbf{y} = (y_1, \ldots, y_n)$ *and* $\mathbf{\Upsilon} = (\Upsilon_1 \ldots, \Upsilon_n)$, *the nominal model* G *and the* a priori *sets* $\mathcal{N}$, $\mathbf{\Delta}$ *determine whether or not the* a priori *and* a posteriori *information are consistent, i.e. whether the consistency set*

$$\mathcal{T}(y, \Upsilon, G_o) = \{(\Delta \in \mathbf{\Delta}, \omega \in \mathcal{N})\colon y = \mathcal{P}_n\left\{\mathcal{F}_\ell[G(\Upsilon), \Delta]\right\} * u + \omega\} \tag{9}$$

is nonempty.

3 Main Results

Consider the lower linear fractional interconnection shown in Figure 1:

$$\begin{aligned} y_k &= \mathcal{P}_n\{P(\Upsilon_k)\} * u_k + \mathcal{P}_n\{Q(\Upsilon_k)\} * \zeta_k + \omega_k \\ \eta_k &= \mathcal{P}_n\{R(\Upsilon_k)\} * u_k + \mathcal{P}_n\{S(\Upsilon_k)\} * \zeta_k \\ \zeta_k &= \mathcal{P}_n\{\Delta\} * \eta_k, \end{aligned} \tag{10}$$

which can be expressed in matrix form as follows:

$$\begin{aligned} T_y &= T_P T_u + T_Q T_\zeta + T_\omega \\ T_\eta &= T_R T_u + T_S T_\zeta \\ T_\zeta &= T_\Delta T_\eta. \end{aligned} \tag{11}$$

The following result shows that the validation problem can be recast in an LMI feasibility form that can be efficiently solved.

Theorem 1 *Given time-domain measurements of the input* $\mathbf{u}$, *the output* $\mathbf{y}$ *and the time–varying parameters* $\mathbf{\Upsilon}$, *the LPV model* $G(\Upsilon)$ *is not invalidated by this experimental information if and only if there exist two vectors* $\zeta = (\zeta_1 \ldots, \zeta_n)$ *and* $\omega = (\omega_1 \ldots, \omega_n)$, *such that:*

$$\begin{gathered} M(\zeta) > 0 \\ \text{and} \\ L(\omega) > 0, \end{gathered} \tag{12}$$

where:

$$M(\zeta) \doteq \begin{bmatrix} X(\zeta) & T_\zeta^T \\ T_\zeta & Y(\zeta) \end{bmatrix}$$
$$X(\zeta) \doteq (T_R T_u)^T T_R T_u + (T_R T_u)^T T_S T_\zeta + T_\zeta^T T_S^T T_R T_u$$
$$Y(\zeta) \doteq \left(\frac{1}{\delta^2} I - T_S^T T_S\right)^{-1}$$
$$\omega \doteq \mathbf{y} - \mathcal{P}_n\{P(\mathbf{\Upsilon})\} * \mathbf{u} + \mathcal{P}_n\{Q(\mathbf{\Upsilon})\} * \zeta, \tag{13}$$

and $L(\omega)$ *is defined as in equation (8).*

Proof: The LPV model $G(\Upsilon)$ is not invalidated by the experimental information $\{\mathbf{u}, \mathbf{y}, \mathbf{\Upsilon}\}$ if there exist a $\Delta \in \mathbf{\Delta}$ and an $\omega \in \mathcal{N}$ such that the equations (10) and (11) hold. From Lemma 1 it can be easily shown that existence of an uncertainty block in $\mathbf{\Delta}$ is equivalent to:

$$T_\zeta^T T_\zeta < \delta^2 T_\eta^T T_\eta. \tag{14}$$

Now, replacing the expression of T_η from (11) in the right-hand side of (14), and reordering terms yields:

$$T_\zeta^T \left(\frac{1}{\delta^2} I - T_S^T T_S\right) T_\zeta < (T_R T_u)^T T_R T_u + + (T_R T_u)^T T_S T_\zeta + T_\zeta^T T_S^T T_R T_u. \tag{15}$$

Using Schur complements and the fact that $\|S(\Upsilon)\|_{\ell_2 \to \ell_2} < \delta^{-1}$ gives the first LMI of the set (12), $M(\zeta) > 0$. The second LMI of (12), $L(\omega) > 0$, is simply obtained by replacing the expression of the noise vector ω from (13) in the definition of $\mathcal{N}$ given in the last equation of (8).

Figure 2: The experimental setup

4 Example

In this section we illustrate the proposed method with a practical example that arises in the context of active vision. The physical system under consideration, shown in Figure 2, consists of a Unisight pan/tilt platform with a BiSight stereo head with Hitachi KP-M1 Cameras and Fujinon H10X11EMPX-31 motorized lenses.

In a previous step we have obtained a model for the system, from the command input (in encoder units) to the head to the position of a given target (in pixels), as a function of a time varying focal length f, and estimated an upper bound on the identification error (see [21] for details on this step). So our goal is to (in)validate this model and refine the error bound, by obtaining a suitable LTI uncertainty following sections 2.2 and 3.

4.1 The Model

For identification purposes, commands were given to the head and lenses using a 10 channel $\delta - \tau$ controller and the image processing required to capture the images and locate the target was performed using a Datacube MaxSPARC S250 hosted by a Sun Ultra workstation. From the identification of the physical system we obtained a nominal model –the block $P(\Upsilon)$ in (8) and (10)–, which has a parametric component that takes into account the dependence of the system on the time varying focal length, and a non-parametric component that accounts for unmodelled dynamics [1]:

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} A_p & 0 \\ 0 & A_{np} \end{bmatrix} x_k + \begin{bmatrix} B_{1p} & B_{2p} \\ 0 & B_{np} \end{bmatrix} \begin{bmatrix} w_k \\ u_k \end{bmatrix} \\ \begin{bmatrix} z_k \\ y_k \end{bmatrix} &= \begin{bmatrix} C_{1p} & 0 \\ C_{2p} & C_{np} \end{bmatrix} x_k + \begin{bmatrix} D_{11p} & D_{12p} \\ D_{21p} & D_{22p} + D_{np} \end{bmatrix} \begin{bmatrix} w_k \\ u_k \end{bmatrix} \\ w_k &= \Upsilon_k z_k, \end{aligned} \tag{16}$$

[1] Due to lack of space, the model presented in this example is a reduced order version of the one obtained in [21]

where:

$$\begin{aligned} A_p &= \begin{bmatrix} 0.3107 & 0.7388 & 0.1119 & -0.1018 \\ -0.7388 & -0.1570 & 0.2073 & -0.1719 \\ -0.1119 & 0.2073 & 0.4131 & 0.6569 \\ -0.1018 & 0.1719 & -0.6569 & -0.2362 \end{bmatrix} \\ B_{1p} &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \\ B_{2p} &= \begin{bmatrix} 0.7594 & 0.5076 & -0.1358 & -0.0983 \end{bmatrix}^T \\ C_{1p} &= \begin{bmatrix} 0.1343 & -0.0897 & 0.0240 & -0.0174 \end{bmatrix} \\ C_{2p} &= \begin{bmatrix} 0.1498 & -0.1001 & 0.0268 & -0.0194 \end{bmatrix} \\ D_{11p} &= 0 \\ D_{12p} &= 0.0356 \\ D_{21p} &= -0.6704 \\ D_{22p} &= 0.0356 \end{aligned} \tag{17}$$

and:

$$\begin{aligned} A_{np} &= 10^{-4} \cdot \begin{bmatrix} -4337 & -4186 & -0.2364 & -0.0240 \\ 4186 & 6531. & -0.2895 & -0.0293 \\ -0.2364 & 0.2895 & -2610 & 4163 \\ 0.0240 & -0.0293 & -4163 & -2202 \end{bmatrix} \\ B_{np} &= \begin{bmatrix} 0.2524 & 0.12344 & -0.1063 \cdot 10^{-5} & 0.1078 \cdot 10^{-6} \end{bmatrix}^T \\ C_{np} &= \begin{bmatrix} 0.0631 & -0.0309 & -0.2657 \cdot 10^{-6} & -0.2694 \cdot 10^{-7} \end{bmatrix} \\ D_{np} &= 0.0108. \end{aligned} \tag{18}$$

4.2 Validation Step

For validation purposes, we have assumed that this nominal model is subject to two different types of unstructured uncertainty Δ –additive and multiplicative–, which leads in the first case to the augmented plant:

$$G_{add}(\Upsilon) \doteq \begin{bmatrix} P(\Upsilon) & I \\ I & 0 \end{bmatrix}, \tag{19}$$

and in the second one to:

$$G_{mult}(\Upsilon) \doteq \begin{bmatrix} P(\Upsilon) & I \\ P(\Upsilon) & 0 \end{bmatrix}. \tag{20}$$

For these particular uncertainty types, the first LMI of the set (12) reduces to:

$$\begin{bmatrix} (T_R T_u)^T T_R T_u & T_\zeta^T \\ T_\zeta & \delta^2 \end{bmatrix} > 0, \tag{21}$$

and therefore it is possible to find the minimum upper bound on the norm of the uncertainty Δ so that the LPV model is not invalidated by the available experimental information. This is desirable from a control oriented perspective, since it leads to less conservative controller designs.

The experimental information considered consists of $Nt = 35$ samples of the time response of the real system y_k to a step input u_k while the time varying parameter Υ_k was allowed to vary between 1.0334 and 0.6205 during the experiment, as is shown in Figure 3.

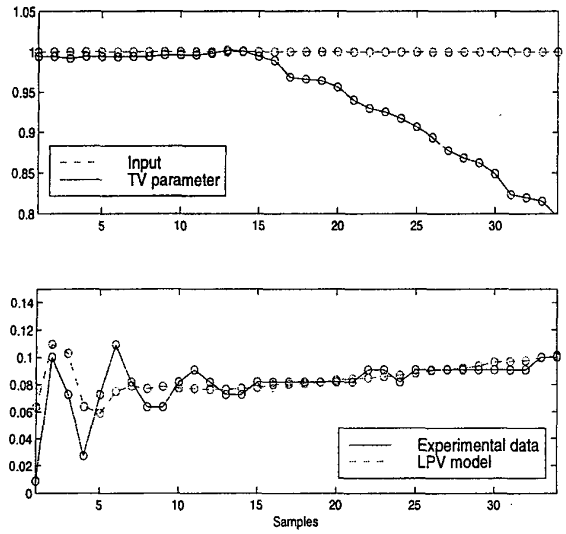

Figure 3: Validation experiment

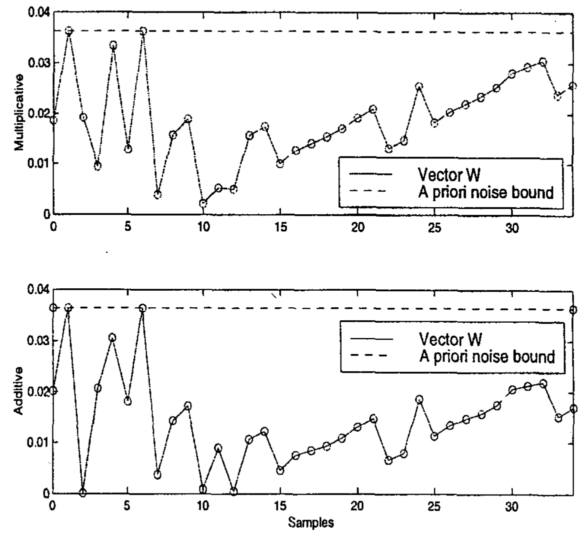

Figure 4: Results of the validation step

By repeatedly measuring the location of the centroid of the target in the absence of input, the experimental noise measurement was determined to be bounded by $\epsilon_t = 4/110$pixels/count [2].

Using this *a priori* information and experimental data, the minimum value of $\|\Delta\|$ such that the LMI (12) holds was determined using Matlab's LMI toolbox to solve the corresponding LMI optimization problem. The LPV model obtained can explain the experimental information, with the sequences of noises plotted in Figure 4 and with the uncertainty block bounded in $\|\cdot\|_\infty$ by $\delta_{add} = 0.0172$ in the additive case and $\delta_{mult} = 0.3456$. Note that the identification was performed taking into account additive uncertainty, which explains the difference between the upper bounds δ_{add} and δ_{mult}.

5 Conclusions and Directions for Further Research

Motivated by the shortcommings of traditional gain–scheduling techniques, during the past few years substantial advances have been made in the problem of synthesizing controllers for Linear Parameter Varying systems. However, the related field of identification of LPV systems is considerably less developed. While tools for robust identification of LPV systems have started to emerge, the problem of LPV model validation has not been addressed yet.

[2]this experimental error is mainly due to fluctuating conditions such as ambient light.

In this paper we propose a new LPV model validation framework that, given experimental data composed by measurements of the output (corrupted by noise) and the time varying parameters, determines whether or not these measurements are consistent with a given plant and uncertainty description. The main result of the paper shows that this problem reduces to an LMI feasibility problem that can be efficiently solved. Thus, it is not more computationally demanding that comparable techniques available for the case of LTI systems.

Efforts are currently under way generalizing these techniques to the cases of structured and time–varying uncertainties.

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