A Model (In)Validation Approach to Gait Recognition

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Abstract

This paper addresses the problem of human gait recognition by applying model (in)validation techniques. The main idea is to associate to each class of gaits a nominal model and a class of bounded energy inputs. In this context, the problem of recognizing a sequence can be formulated as the problem of determining whether or not it could have been generated by a given model and its associated class of inputs. By exploiting interpolation theory results this problem can be recast into a Linear Matrix Inequality (LMI) optimization form and efficiently solved.

1. Introduction

This paper addresses the problem of recognizing three different types of human gaits, namely walking, running and walking a staircase, by applying model (in)validation techniques (see [3, 6] for a survey).

The experimental data consist of measurements of the angles of the shoulder, elbow, hip and knee joints of a person walking, running or walking a staircase. Following [1], these sequences are assumed to be realizations of a second order stationary stochastic process and hence can be associated to a causal, discrete-time, linear time-invariant system driven by white noise. These models can be obtained for instance by using subspace identification methods (see [7]). In [1] it has been proposed that a given sequence can be recognized by finding its associated model and then finding its closest neighbour, in the sense of the Martin distance (see [1] and references therein), among the set of known gaits.

This paper takes a different approach towards gait recognition. The idea is to associate to each class of gaits a nominal model and a class of inputs of bounded energy. These nominal models can be obtained from the training data by finding, in each class, the model that is *closest* to each other element in some sense. In this context, the problem of determining whether or not a given experimental sequence corresponds to a particular gait type can be formulated as a model (in)validation problem.

The paper is organized as follows. Section 2 introduces the notation and required results. Section 3 states the problem of gait recognition as a model (in)validation one. Section 4 shows that the problem above can indeed be recast as a Linear Matrix Inequality optimization problem by invoking Carathéodory-Fejér interpolation theory, and efficiently solved. Section 5 illustrates the proposed technique with a practical example. Finally, Section 6 presents the conclusions and possible directions for future research.

2. Preliminaries

Let $\mathbf{x} \in \mathbf{R}^m$ denote a column vector and $\|\mathbf{x}\|_2$ its euclidean norm. Let $_2^n$ be the space of real, one sided, square summable, finite vector sequences $x \doteq \{\mathbf{x}_i\}_{i=0}^n$ equipped with the norm:

$$\|x\|_{\ _2^n} \doteq \ \sum_{i=0}^{n} \|\mathbf{x}_i\|_2^2 \ ^{\frac{1}{2}}, \tag{1}$$

and let $_2$ denote its extension to infinite length sequences. For any sequence $x \in \ _2^n$, define the following finite lower Toeplitz matrix:

$$\mathbf{T}_x^n = \begin{matrix} \mathbf{x}_0 & 0 & \dots & 0 \\ \mathbf{x}_1 & \mathbf{x}_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_{n-1} & \mathbf{x}_{n-2} & \dots & \mathbf{x}_0 \end{matrix} . \tag{2}$$

Let $\mathcal{H}_{\infty,\rho}$ denote the space of complex-valued matrix functions essentially bounded on $|z| = \rho$ and with bounded analytic continuation in $|z| \ \ \rho$, equipped with the norm:

$$\|L\|_{\infty,\rho} \doteq ess \sup_{|z| \ \ \rho} \overline{\sigma}(L(z)) \tag{3}$$

where $\overline{\sigma}$ represents the largest singular value, and let $\mathcal{BH}_{\infty,\rho}(\delta)$ denote the closed δ-ball in $\mathcal{H}_{\infty,\rho}$:

$$\mathcal{BH}_{\infty,\rho}(\delta) = \{L \in \mathcal{H}_{\infty,\rho} : \|L\|_{\infty,\rho} \le \delta\}. \tag{4}$$

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In the sequel, $\mathcal{H}_\infty$ and $\|.\|_\infty$ stand for the case $\rho = 1$, and $\mathcal{BH}_\infty$ for the closed unit ball in $\mathcal{H}_\infty$.

This paper considers finite-dimensional, causal, discrete-time, LTI systems bounded in $_2$, i.e.

$$\|S\|_{\ 2\to\ 2} \doteq \sup_{\|u\|_{\ 2}=0} \frac{\|S\ \ u\|_{\ 2}}{\|u\|_{\ 2}} \quad \infty, \tag{5}$$

where stands for convolution. Any such system will be represented by an infinite lower Toeplitz matrix $\mathbf{T}_S$ mapping input to output sequences in $_2$:

$$\begin{matrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \end{matrix} = \begin{matrix} \mathbf{S}_0 & 0 & 0 & \\ \mathbf{S}_1 & \mathbf{S}_0 & 0 & \cdots \\ \mathbf{S}_2 & \mathbf{S}_1 & \mathbf{S}_0 & \\ & \vdots & & \ddots \end{matrix} \begin{matrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \end{matrix}, \tag{6}$$

where $\{\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2, \cdots\}$ is its convolution kernel, or by the finite upper left submatrix of $n \times n$, $\mathbf{T}_S^n$, obtained from the infinite matrix above, when dealing with finite input-output sequences in $^n_2$. Alternatively, any system of interest will be represented by a minimal (not unique) state-space realization, or by its $\mathcal{Z}$-transform evaluated at $1/z$, i.e. $S(z) = \sum_{k=0}^{\infty} \mathbf{S}_k z^k$, $S(z) \in \mathcal{H}_{\infty,\rho}$, for some $\rho > 1$. It is a well known fact that in this case $\|S\|_{\ 2\to\ 2} = \sup_{|z|=1} \overline{\sigma}(S(z))$.

Finally, for a real matrix $\mathbf{A} \in \mathbf{R}^{m\times n}$, $\mathbf{A}^T$ denotes its transpose, i.e. $\mathbf{A}^T \in \mathbf{R}^{n\times m}$; for a real square symmetric matrix $\mathbf{A} = \mathbf{A}^T \in \mathbf{R}^{m\times m}$, $\mathbf{A} > 0$ means that $\mathbf{A}$ is positive definite, i.e.

$$\mathbf{x}^T \mathbf{A} \mathbf{x} > 0 \quad \forall \mathbf{x} \in \mathbf{R}^m, \mathbf{x} = \mathbf{0}, \tag{7}$$

$\mathbf{A}$ 0 that $\mathbf{A}$ is positive semidefinite and $\mathbf{A}$ 0 that $\mathbf{A}$ is negative definite.

The following result will be required to solve the gait recognition problem as a model (in)validation one.

Lemma 1 (Carathéodory-Fejér) *Given a matrix valued sequence $\{\mathbf{L}_i\}_{i=0}^{n-1}$, there exists a causal, discrete-time, LTI operator $L(z) \in \mathcal{BH}_\infty$ such that*

$$L(z) = \mathbf{L}_0 + \mathbf{L}_1 z + \mathbf{L}_2 z^2 + \cdots + \mathbf{L}_{n-1} z^{n-1} + \cdots \tag{8}$$

if and only if

$$(\mathbf{T}_L^n)^T \mathbf{T}_L^n \leq \mathbf{I} \tag{9}$$

where $\mathbf{I}$ denotes the identity matrix of compatible dimension.

Proof: See for instance [4], Chapter 1. □

3. Problem Statement

Consider the gait recognition set-up depicted in Figure 1. Here, S is a causal, discrete-time, LTI model driven by white zero-mean Gaussian noise (see [7] for details):

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{K}\mathbf{e}_k, \quad \hat{\mathbf{y}}_k^S = \mathbf{C}\mathbf{x}_k + \mathbf{e}_k \tag{10}$$

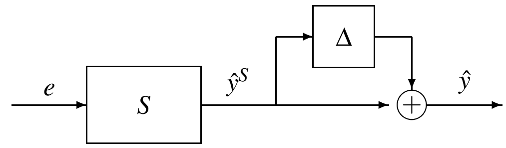

Figure 1. The Gait Recognition Set-up

which is assumed to be representative of a particular gait type. Also by assumption:

$$\hat{\mathbf{y}}_k^S \doteq \mathbf{y}_k^S - \mathrm{E}(\mathbf{y}_k^S), \quad \mathrm{E}(\mathbf{y}_k^S) = \boldsymbol{\mu} \ \forall k, \tag{11}$$

where E denotes expected value[1] and $\mathbf{y}_k^S$ is a vector with measurements of the angles of the shoulder, elbow, hip and knee joints of a person walking, running or walking a staircase, at instant k, used to get nominal model S.

The goal is to decide if a given experimental sequence $\hat{y}$ –different from $\hat{y}^S$– belongs to the gait type represented by model S. The mismatch between sequences $\hat{y}^S$ and $\hat{y}$ is modelled by:

$$\hat{y} = (I + \Delta) S \quad e, \tag{12}$$

with I the identity operator, Δ a bounded operator in $\mathcal{BH}_\infty(\delta)$, $\delta \quad 1$ [2]. and e a possible input sequence of bounded energy over the finite horizon $0, n$ in the set:

$$\mathcal{U} = \{e\colon \sum_{k=1}^{N} \mathbf{e}^T \mathbf{e} \leq \varepsilon^2\}. \tag{13}$$

In this context, the gait recognition problem is equivalent to determine whether model S could have generated the sequence $\hat{y}$. This is precisely a model (in)validation problem.

Problem 1 *Given a nominal model for a given gait type $\mathcal{S}$ as in (10), sets $\mathcal{U}$ and $\boldsymbol{\Delta}$ of possible inputs and uncertainty blocks:*

$$\mathcal{U} = \{\mathbf{e}\colon \sum_{k=1}^{N} \mathbf{e}^T \mathbf{e} \leq \varepsilon^2\}, \quad \boldsymbol{\Delta} = \{\Delta\colon \Delta \in \mathcal{BH}_\infty(\delta), \delta \quad 1\}$$

and the experimental sequence $\hat{y}$, determine whether or not there exists at least one feasible pair $(\mathbf{e}, \Delta)$ in the sets $\mathcal{U}$, $\boldsymbol{\Delta}$ so that model $\mathcal{S}$ can reproduce the available experimental evidence.

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[1] For practical purposes, it will be additionaly required that process y is mean-ergodic. Under this assumption it is possible to get an estimate of $\boldsymbol{\mu}$ from temporal averages over the finite horizon $0, n$ with n sufficiently large, which is guaranteed to converge as $n \to \infty$ to the true $\boldsymbol{\mu}$ in the mean square sense (see for instance [5], Chapter 13).

[2] This requirement is imposed for the problem to make sense, i.e. $\|\hat{y} - S\ \ e\|_{\ ^n_2} \leq \|S\ \ e\|_{\ ^n_2}$.

If the answer to Problem 1 is negative the experimental sequence does not correspond to the assumed gait type. On the other hand, if such a feasible pair exists it is clearly of interest to quantify the minimum size (in the $\mathcal{H}_\infty$ sense) of the uncertainty block.

Therefore, by selecting representative models of each gait type and finding the minimum size of the uncertainty block that yiels an afirmative answer to Problem 1, it is possible to assign the unknown gait to one of the given classes.

4. Main Results

Next Theorem shows that Problem 1 can be recast as an LMI feasibility problem, by invoking Carathéodory-Fejér interpolation theory.

Theorem 1 *Problem 1 has an affirmative answer if and only if there exists a finite sequence $e = \{\mathbf{e}_0, \mathbf{e}_1, \ldots, \mathbf{e}_n\}$ so that the following set of LMIs hold:*

$$\begin{aligned} \mathbf{A}_1(e) &\doteq \begin{matrix} \mathbf{X}(e) & (\mathbf{T}_S^n \mathbf{T}_e^n)^T \\ \mathbf{T}_S^n \mathbf{T}_e^n & (\delta^2 - 1)^{-1}\mathbf{I} \end{matrix} \leq 0 \\ \mathbf{A}_2(e) &\doteq \begin{matrix} \varepsilon^2 & \mathbf{Y}^T(e) \\ \mathbf{Y}(e) & \mathbf{I} \end{matrix} \quad\quad 0 \end{aligned} \tag{14}$$

with:

$$\begin{aligned} \mathbf{X}(e) &\doteq (\mathbf{T}_{\hat{y}}^n)^T \mathbf{T}_{\hat{y}}^n - (\mathbf{T}_{\hat{y}}^n)^T \mathbf{T}_S^n \mathbf{T}_e^n - (\mathbf{T}_S^n \mathbf{T}_e^n)^T \mathbf{T}_{\hat{y}}^n \\ \mathbf{Y}(e) &\doteq \begin{matrix} \mathbf{e}_0^T & \mathbf{e}_1^T & \cdots & \mathbf{e}_n^T \end{matrix}^T, \end{aligned}$$

$\mathbf{I}$ *the identity matrix of compatible dimension, and $\mathbf{T}_{\hat{y}}^n$, $\mathbf{T}_e^n$ and $\mathbf{T}_S^n$ defined as in Section 2.*

Proof: According to Figure 1,

$$\mathbf{T}_{\hat{y}^S}^n = \mathbf{T}_S^n \mathbf{T}_e^n, \quad \mathbf{T}_z^n = \mathbf{T}_\Delta^n \mathbf{T}_{\hat{y}^S}^n, \quad \mathbf{T}_{\hat{y}}^n = \mathbf{T}_z^n + \mathbf{T}_{\hat{y}^S}^n \tag{15}$$

where z represents the output sequence from the uncertainty block Δ. Clearly, model S could have generated the experimental evidence $\hat{y}$ if and only if there exists a pair (Δ, e) satisfying equations (15). As a consequence of Lemma 1, there exists a $\Delta \in \mathbf{\Delta}$ mapping the input-output sequences $(\hat{y}^S, z)$ if and only if

$$(\mathbf{T}_z^n)^T \mathbf{T}_z^n \leq \delta^2 (\mathbf{T}_{\hat{y}^S}^n)^T \mathbf{T}_{\hat{y}^S}^n. \tag{16}$$

Combining equations (15) and reordering terms yields:

$$\begin{aligned} (\mathbf{T}_{\hat{y}}^n)^T \mathbf{T}_{\hat{y}}^n - (\mathbf{T}_{\hat{y}}^n)^T \mathbf{T}_S^n \mathbf{T}_e^n - (\mathbf{T}_S^n \mathbf{T}_e^n)^T \mathbf{T}_{\hat{y}}^n \\ - (\delta^2 - 1)(\mathbf{T}_S^n \mathbf{T}_e^n)^T \mathbf{T}_S^n \mathbf{T}_e^n \leq 0. \end{aligned} \tag{17}$$

Noticing that by assumption $\delta \quad 1$ and using Schur complements (see [2], Chapter 2, and references therein) gives the first LMI in (14). The second LMI is a simple restatement of (13), by invoking a Schur complement argument. □

Person	Walking	Running	Staircase
A	1,2	16 to 18	25 to 27
B	3 to 8	11 to 15	21 to 24
C	9,10	none	28 to 30
D	none	19	none
E	none	20	none

Figure 2. Experimental Data

Note that since $\alpha(\delta) \doteq (1 - \delta^2)^{-1}$ for $\delta \in (0, 1)$ is a convex function of δ, it is possible to optimize over the size of the uncertainty required to explain the data, by solving the following problem:

$$\min \alpha, \quad \text{so that:} \quad \hat{\mathbf{A}}_1(e, \alpha) \leq 0, \quad \mathbf{A}_2(e) \quad 0, \quad \alpha > 1$$

where $\hat{\mathbf{A}}_1(e, \alpha)$ results from replacing block $(2, 2)$ in $\mathbf{A}_1(e)$ by $-\alpha\mathbf{I}$.

5. Example

This example begins by outlining a method to compute suitable nominal models for each gait class, from a training set of sequences. Then, using *new* experimental sequences of different human beings, it illustrates the proposed method for gait recognition.

The experimental data. The experimental data consists of 30 vector sequences, taken from 5 different persons, named A, B, C, D and E. Each sequence contains measurements of the angles of the shoulder, elbow, hip and knee joints of a person walking, running or walking a staircase. For illustrative sake, these sequences are numbered from 1 to 30 so that the first 10 correspond to walking, the second set of 10 to running and the third set of 10 to walking a staircase, as shown in Table 2.

The nominal models. Let S_i denote any candidate model as in (10), of 4 states, 4 inputs and 4 outputs, associated to the experimental sequence y_i, as introduced in Section 3. Thus for recognition purposes, S_i must be specified together with mean vector $\boldsymbol{\mu}_i = \mathrm{E}(\mathbf{y}_{i_k})$ and an upper bound on the input energy ε_i. In this paper, the mean of a given sequence will be estimated as $\sum_{k=0}^{m} \mathbf{y}_{i_k}/(m+1)$ and ε_i will be computed as the input energy required for model S_i to generate y_i, i.e. $\varepsilon_i \doteq \|e\|_{\ell_2^n} : e = S_i^{-1} \; y_i$. Given a particular gait type and a set of models $\mathcal{S}$, computed from a training set of sequences, define the nominal model $S \in \mathcal{S}$ as the one that is closest to each other element in its class, in the sense of minimizing the norm of the (multiplicative) uncertainty required to map the two models under consideration, i.e

$$S = \arg \min_{\hat{S}_i, \hat{S}_j \in \mathcal{S}} \quad \|(\hat{S}_i - \hat{S}_j)\hat{S}_j^{-1}\|_\infty \quad , \tag{18}$$

in accordance to (12), where $\hat{S}_i \doteq \varepsilon_i S_i$ [3]. Proceeding as described above yields the following three nominal models, denoted as S_{walk} for walking, S_{run} for running and S_{stair} for walking a staircase:

$$S_{walk} \doteq S_{10}, \quad S_{run} \doteq S_{20}, \quad S_{stair} \doteq S_{30}. \tag{19}$$

Thus, sequences $\{y_{10}, y_{20}, y_{30}\}$ are the training data for the problem. Let's apply the model (in)validation framework presented in Sections 3 and 4 to the remaining experimental sequences.

The results. Table 3 shows the results of applying Theorem 1, using 20 sample points per sequence. In all cases, the first column contains the experimental sequences to be recognized; the second, third and fourth columns display the minimum size of the uncertainty block Δ measured in the $\mathcal{H}_\infty$ norm, so that nominal models S_{walk}, S_{run} and S_{stair} can reproduce the given data. Notice that by assumption, all norms are no greater than 1; $\|\Delta\|_\infty = 1$ means that the assumed model is invalidated by the data, leading to the conclusion that the given sequence cannot correspond to that gait. Thus by examining each row and selecting the smallest uncertainty norm (indicated by a †), all sequences can be assigned to a particular gait type. As can be seen from Tables 2 and 3, the proposed method can succesfully recognize 25 from the 27 sequences under consideration; it only confuses 2 sequences –y_{26} and y_{29}, belonging to persons A and C walking a staircase– as walking sequences. The failure could be attributed to the length of the experiment used for recognition purposes, or simply to faulty sequences, specially because the proposed method is able to correctly recognize sequences $\{y_{25}, y_{27}\}$ and y_{28} from A and C respectively.

Sequence	S_{walk}	S_{run}	S_{stair}
y_1	0.1743†	0.6758	0.5973
y_2	0.2333†	0.5818	0.2427
y_3	0.0305†	0.6843	0.6866
y_4	0.0410†	0.6217	0.5305
y_5	0.0819†	0.6915	0.6069
y_6	0.0001†	0.6879	0.7688
y_7	0.0900†	0.6892	0.9188
y_8	0.2068†	0.6037	0.7883
y_9	0.0001†	0.6926	0.6028
y_{11}	0.9265	0.3415†	1.0000
y_{12}	0.9676	0.2452†	0.9325
y_{13}	1	0.0002†	0.9323
y_{14}	1	0.0002†	0.9903
y_{15}	1	0.0002†	0.8999
y_{16}	1	0.0005†	0.5707
y_{17}	0.9220	0.0532†	0.5437
y_{18}	1	0.0004†	0.6961
y_{19}	1	0.3545†	0.8374
y_{21}	0.9631	0.5002	0.3174†
y_{22}	0.7952	0.4122	0.0577†
y_{23}	0.7215	0.4089	0.0936†
y_{24}	0.8499	0.4456	0.0805†
y_{25}	0.7252	0.5928	0.3962†
y_{26}	0.6828†	0.7127	0.8827
y_{27}	0.5553	0.5818	0.4682†
y_{28}	0.2650	0.6801	0.1699†
y_{29}	0.0391†	0.6102	0.1470

Figure 3. Gait Recognition Results

6. Conclusions

This paper approaches the problem of human gait recognition from a model (in)validation viewpoint. The proposed method, which involves comparing any given experimental sequence against a fixed set of nominal models for each gait type, is shown to be succesful by means of a practical example. Issues such as model and uncertainty structure, capable of extracting more information from the available experimental evidence, remain open for future research.

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[3]The scaling is required to make models comparable in the context of Problem 1. This means solving the problem by looking for a feasible input sequence in the unit ball $\|e\|_{\ell_2^n} \leq 1$.

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