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Step response bounds for time varying uncertain systems with bounded disturbances

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Abstract

In this paper we focus our attention on the determination of upper bounds of the l^∞ norm of the output of a linear discrete-time dynamic system driven by a step input, in the presence of both persistent unknown-but-l^∞ bounded disturbances and memoryless time-varying model uncertainty. For the same type of systems we also analyze the transient behavior of the step response in terms of its overshoot. The problem is solved in a constructive way by determining appropriate D-invariant sets contained in a given convex region. Finally, we show how to extend these results to continuous-time systems.

1. Introduction

In most practical situations the mathematical model of a dynamic system must include some uncertainties and disturbances due to unmodeled dynamics and/or time varying conditions. In this paper we investigate the problem of robust performance (in the l^∞ sense) of dynamic systems subject to parametric time-varying uncertainties and in the presence of l^∞ bounded disturbances. The problem of interest is to determine a bound on the worst-case l^∞ norm of the output due to a step input, and with zero initial conditions. We are also interested in checking, similarly to what happens when no uncertainties are present, if the system presents an overshoot with respect to its steady state output value.

In [5] robust performance conditions with respect to unknown but bounded disturbances are provided. However, in many real problems, some design specifications are given in terms of the output to a given, fixed test signal (such as a step). In principle this problem can be addressed using the techniques proposed in [5]. However, this approach will yield a conservative bound, since these results provide the worst case l^∞ bound of the output over the set of all possible l^∞ bounded inputs.

The problem of robust performance under structured operator (dynamical) uncertainty blocks has been addressed in [11], where necessary and sufficient conditions for robust steady state tracking have been provided and in [9], where lower and upper bounds for the maximum overshoot are given.

In this paper we provide a nonconservative bound for the case where the input signal is known. This bound is obtained using a method based upon the construction of a suitable polyhedral region. Using the same construction we also provide necessary and sufficient for the existence of overshoot and a way to compute both the steady state output value and the overshoot, when present.

2. Preliminaries

2.1. Notation

Given a closed, convex set S we denote its interior as $int\{S\}$. A polyhedral set S will be represented by a set of linear inequalities $S = \{x : F_i x \leq g_i, i = 1\dots, s\}$, as well as by its dual representation $S = \{conv(x_j)\}$ in terms of its vertex set $\{x_i\}$, which will be denoted by $vert\{S\}$. In the sequel we will use matrix compact notation to describe componentwise assignments as well as componentwise inequalities. Thus, in this notation a polyhedral set is expressed by the matrix inequality $S = \{x : Fx \leq g\}$ where F is a $s \times n$ full column rank matrix and g represents an s-column vector. Finally $d(x, S)$ will denote the distance between a point x and a set S, computed as $d(x, S) = \inf_{y \in S} \|x - y\|$.

2.2. Problem statement

Consider the uncertain n-dimensional discrete time system with m command inputs $u(k)$, q disturbance inputs $d(k)$ and p outputs:

$$\begin{aligned} x(k+1) &= A(w(k))x(k) + Bu(k) + Ed(k) \\ y(k) &= Cx(k) \end{aligned} \quad (1)$$

where $w(k)$ is an uncertain time-varying parameter, $A(w)$ is a matrix polytope of the form

$$\begin{aligned} A(w) &= \textstyle\sum_{i=1}^r A_i w_i(k), \\ w(k) \in W &= \{w : w_i \geq 0, \textstyle\sum_{i=1}^r w_i = 1\}, \end{aligned} \quad (2)$$

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where A_i, B, E are given real matrices of appropriate dimensions and where the disturbance $d(k)$ belongs to the l^∞ unity ball, i.e.: $||d(k)||_{l^\infty} \leq 1$.

For these systems we are interested in determining a non conservative bound of the output of the system step response, i.e. the problem we address is the following:

Problem 1 *Given the system (1) with zero initial state, and a step input of the form $u(k) = U, k \geq 0$, find:*

$$\mu_{inf} = \left\{ \begin{array}{l} \inf \mu : ||y(k)||_{l^\infty} \leq \mu \text{ for all} \\ w(k) \in W, d(k),\ ||d||_{l^\infty} \leq 1 \end{array} \right\} \quad (3)$$

In order to simplify the exposition we make the following assumption.

Assumption 1 *There exists a matrix A_0 belonging to the matrix polytope such that the triplets (A_0, B, C) and (A_0, E, C) are reachable and observable.*

Under assumption 1 it easily shown that a necessary and sufficient condition for Problem 1 to have a finite solution $\mu < +\infty$ is that the autonomous system $x(k+1) = A(w(k))x(k)$ is asymptotically stable. Thus, in the sequel we will limit our attention to asymptotically stable systems.

Definition 2.1 *Consider the system $x(k+1) = A(w(k))x(k) + E^*d^*(k)$, where $d^*(k) \in D^*$ a convex and compact set. A closed and convex set P is positively D^*-invariant (D^*-invariant for brevity) for this system if for every initial condition $x(0) \in P$ we have that $x(k) \in P$ for every $k \geq 0$, for every admissible disturbance $d(k) \in D^*$ and every sequence $w(k)$ as in (2).*

Given a convex set C define $I(C)$ as the class of all D^*-invariant sets included in C. If S_1 and S_2 are in $I(C)$ then the following properties hold

- **i)** their intersection $S_1 \bigcap S_2$ is in $I(C)$;
- **ii)** their convex hull $conv\{S_1 \bigcup S_2\}$ is in $I(C)$.

This means that the class of $I(C)$ together with the operations of intersection and convex hull is a lattice. Hence it admits a supremum and an infimum, that is there exist a maximal and a minimal D^*-invariant set in $I(C)$. Therefore given a convex set C the "largest" and "smallest" D^*-invariant sets (i.e. the D^*-invariant set which contains any D^*-invariant subset of C and respectively the one which is included in any D^*-invariant subset of C) are both well defined.

3. Main Results

3.1. Limit set

Let us now introduce an "extended disturbances" system which treats the command inputs of system (1) as disturbances:

$$\begin{array}{rcl} x(k+1) & = & A(w(k))x(k) + E^*d^*(k) \\ y(k) & = & Cx(k) \end{array} \quad (4)$$

where $E^* = [B\ E]$, $d^*(k) = [u^T(k)\ d^T(k)]^T$ and the extended disturbance $d^*(k)$ is constrained to belong to the polyhedral set

$$D^* = \{[u^T(k)\ d^T(k)]^T : u(k) = U,\ ||d(k)||_{l^\infty} \leq 1\}.$$

Definition 3.1 *Given a dynamic system as in (4), the limit set $\mathcal{L}$ is defined as the set of all the states x for which there exist some sequences w, d^* and t_k such that*

$$\lim_{k \to +\infty} \phi(0, t_k, w(\cdot), d^*(\cdot)) = x$$

where $\lim_{k \to +\infty} t_k = +\infty$ and $\phi(0, t_k, w(\cdot), d^*(\cdot))$ denotes the trajectory of system (4) originating at $x_0 = 0$ and corresponding to w and d^*.

Lemma 3.1 *If system (1) is asymptotically stable then the state evolution of system (4), for every possible disturbance $d^*(k) \in D^*$ and every sequence $w(k)$, converges to the limit set $\mathcal{L}$ which is compact and is the minimal D^*-invariant set for system (4).*

Define now the set:

$$X_0(\mu) = \{x : ||Cx||_\infty \leq \mu\}. \quad (5)$$

A value $\mu < +\infty$ is admissible if $\mu > \mu_{inf}$. Clearly a *necessary* condition for μ to be admissible is that the set $\mathcal{L}$ is contained in the region $X_0(\mu)$. This condition is not sufficient because even if $\mathcal{L} \subset X_0(\mu)$ there may be trajectories starting from the origin that leave the region $X_0(\mu)$ and ultimately enter in it to reach $\mathcal{L}$. Thus the knowledge of $\mathcal{L}$ does not give enough information to assess the complete system behavior. So, in principle, to compute the maximum overshoot, one should reconstruct all the possible trajectories originating from the origin. This could be done by propagating forward in time the effect of the uncertainties as shown in [1]. Denote by R_k the set of all states that can be reached in k steps from the origin for all admissible w and d. As noticed in [1] the propagation of the uncertainties effect forward in time produces non-convex reachability sets R_k. However, it can be shown with the same technique used in [1] that, denoting by $\tilde{R}_k$ the convex hull of R_k, the sequence of convex sets $\tilde{R}_k$ can be generated recursively. It can be shown that $\tilde{R}_k$ "converges" to $\tilde{\mathcal{L}}$, the convex hull of $\mathcal{L}$. It is immediate to verify that the system has a performance less or equal to μ if and only if $\tilde{R}_k \subset X_0(\mu)$, for all $k > 0$. However, proceeding in this way might be not realistic because of the computational effort necessary to compute $\tilde{R}_k$ and because there is no a reasonable stopping criterion for the procedure (i.e. how many $\tilde{R}_k$ to compute).

We will solve this problem proceeding in a different way, leading to a condition related to a **single** convex set. We state now the basic result of this section which will be used to give a solution to Problem 1.

Lemma 3.2 *Given $\mu > 0$, the response of the system (1) to the input $u(k) = U$ satisfies $||y||_{l^\infty} \leq \mu$ for every pair of sequences $w(k)$, $d^*(k) \in D^*$ if and only if the maximal D^*-invariant set for system (4) contained in $X_0(\mu)$ contains the origin.*

3.2. Computation of the maximal D*-invariant set

In this section we provide a procedure to compute the maximal D^*-invariant set for system (4).

Given a compact set S, we can define the preimage set $C(S)$ of S as the set of all the states x which, under the mapping $A(w)x + E^*d^*$, are mapped in S for all $d^* \in D^*$ and w as in (2). If the set S is polyhedral it can be represented as $S = \{x : Fx \leq g\}$, hence $C(S)$ can be expressed as

$$C(S) = \{ \quad x: \ F(A(w)x + E^*d^*) \leq g, \text{ for all } d^* \in D^* \text{ and } w \text{ as in (2)}\} \tag{6}$$

As the set D^* is itself polyhedral, the set $C(S)$ is defined by the following inequalities [3]

$$C(S) = \{x: \ FA_ix \leq g - \delta, \quad i = 1, \dots, r\},$$

where the vector δ has components

$$\delta_j = \max_{d^* \in D^*} F_j E^* d^*.$$

By recursively defining the sets $P^{(k)}$, $k = 0, 1 \dots$ as

$$P^{(0)} = X_0(\mu), \quad P^{(k)} = C(P^{(k-1)}) \bigcap P^{(k-1)},$$

we have that $P^{(\infty)}$ is the maximal D^*-invariant set contained in $X_0(\mu)$.

We now introduce a theorem which guarantees that the maximal D^*-invariant set contained in $X_0(\mu)$ can be expressed by a set of linear inequalities (i.e it is polyhedral) and thus can be finitely determined.

Theorem 3.1 *Suppose that system (4) is asymptotically stable. Then, if $\mathcal{L}$ is contained in the interior of $X_0(\mu)$ for some $\mu > 0$, the maximal D^*-invariant set contained in $X_0(\mu)$ is polyhedral. Moreover in this case there exists k^* such that $P^{(\infty)} = P^{k^*}$ and this k^* can be selected as the smallest integer such that $P^{(k)}$ satisfies the vertex condition*

$$A(w)x_j + E^*d_i^* \in P^{(k)} \tag{7}$$

for every $x_j \in vert\{P^{(k)}\}$ and $d_i^* \in vert\{D^*\}$.

Proof. Let $P_0^{(\infty)}$ denote the largest invariant set contained in $X_0(\mu)$ for the system $x(t+1) = A_0x(t) + BU$. Since $P_0^{(\infty)}$ is compact [18] and since the maximal D^*-invariant set contained in $X_0(\mu)$ is included in $P_0^{(\infty)}$, it follows that hence $P^{(\infty)}$ will be equal to the maximal invariant set contained in any compact polyhedral set S containing $P_0(\infty)$ and contained in $X_0(\mu)$. Now from the stability of (4) we have that the state trajectories converge exponentially to $\mathcal{L} \subset int\{X_0(\mu)\}$. Thus, proceeding as in [5], it can be shown that there exists k^* such that $P^{(k^*+1)} = P^{(k^*)} = P^{(\infty)}$. Finally, the proof of (7) can be found in [2]. □

To solve Problem 1 we determine the maximal D^*-invariant set contained in $X_0(\mu)$ [1] defined as in (5) for several values of μ and we check if this set contains the origin. Then

- If $\mu_{inf} < \mu$ we get a positive answer
- If $\mu_{inf} > \mu$ we get a negative answer.

Note that in both cases we get an answer *in a finite number of steps.* In the first case this is due to Theorem 3.1. In the second case, this follows by the fact that the sequence of closed sets $P^{(k)}$ is ordered by inclusion and $P^{(\infty)}$ is their intersection. Thus $0 \notin P^{(\infty)}$ if and only if $0 \notin P^{(k)}$ for some k. A further negative answer can be derived by the following theorem. This negative criterion will become fundamental for the overshoot problem in the next section.

Theorem 3.2 *If the set $P^{(k)}$ is contained in the interior of $X_0(\mu)$ for some k, then the system (4) does not admit a D^*-invariant set contained in $X_0(\mu)$.*

Proof. Suppose there exists k such that $P^{(k)}$ is contained in the interior of $X_0(\mu)$ and system (4) admits an invariant region, hence a maximal one $P^{(\infty)}$, contained in $int\{X_0(\mu)\}$. Define the quantity ν as

$$0 < \nu \doteq \inf_{x \notin X_0(\mu)} d(x, P^{(\infty)})$$

For every initial condition $x_0 \notin P^{(\infty)}$ there exist sequences $\bar{w}$ and $\bar{d}^*$ such that the corresponding trajectory satisfies

$$\sup_{k \to \infty} ||Cx(k)||_\infty = \bar{\mu} > \mu$$

and hence there exists $\bar{k}$ such that $x(\bar{k})$ doesn't belong to $X_0(\mu)$. If we compute the system evolution starting from x_0 and $y_0 \in P^{(\infty)}$, for the same sequences $\bar{w}$ and $\bar{d}^*$, we have that the updating equation for the difference $e(k) = x(k) - y(k)$ is described by

$$e(k+1) = A(\bar{w}(k))e(k) \tag{8}$$

[1]This set can be obtained by starting from the initial set $X_0(\mu)$ and proceeding backwards to compute the sequence of sets $P^{(k)}$ until some appropriate stopping criteria are met.

Now $x(\bar{k}) \notin X_0(\mu)$ means that $\|e(\bar{k})\|_\infty \geq \nu$, but the relation (8) is linear and the system is stable and hence, for every $\delta > 0$ there exists $\epsilon > 0$ such that, for every $x_0 \notin P^{(\infty)}$, with $\|x_0 - y_0\|_\infty < \epsilon$, we have $\|x(\bar{k}) - y(\bar{k}))\|_\infty \leq \delta$. □

These results allows us to derive the following constructive procedure to find a robust performance bound:

Procedure 3.1 *The problem data are the system matrices, the input amplitudes U, the disturbance set D and a test output bound μ*

0 Set $k = 0$ and set $P^{(0)} = X_0(\mu) = \{x : F^{(0)}x \leq g^{(0)}\}$.

1 Consider the set $Q^{(k)} = \{x : F^{(k)}A_i x \leq g^{(k)} - \delta^{(k)},\ \ i = 1,\ldots,r\}$, where the vector $\delta^{(k)}$ has components $\delta^{(k)} = \max_{d\in D^*} F^{(k)}E^*d$.

2 Compute the set $P^{(k+1)} = Q^{(k)} \bigcap P^{(k)}$.

3 If $0 \notin P^{(k+1)}$ or $P^{(k+1)} \subset int\{X_0(\mu)\}$ then stop, the procedure has failed. Thus the output does not robustly meet the performance level μ.

4 If $P^{(k+1)}$ satisfies the vertices condition (7) stop (this implies $P^{(k+1)} = P^\infty$ the maximal D^*-invariant set).

5 Set $k = k + 1$ and go to step 1.

This procedure can then be used together with a bisection method on μ to approximate arbitrarily close the optimal value μ_{inf}, that solves Problem 1. In fact if the procedure stops at step 3 we conclude that $\mu < \mu_{inf}$ and we can increase the value of the output bound μ. Else, if the procedure stops at step 4, we have determined an admissible bound for the output, say $\mu > \mu_{inf}$, that can be decreased. The procedure may fail to converge for the value $\mu = \mu_{inf}$. However, it can be shown that for any value of $\mu \neq \mu_{inf}$ the procedure terminates in a finite number of steps. Nevertheless, to avoid the possibility of an endless loop we might put a bound on the maximum admissible number of iterations.

3.3. The overshoot and steady-state problem

We have seen that an arbitrarily good approximations of the l^∞ norm of the output of system (1), when driven by a step input of the form $u(k) = U$, can be derived by checking the existence of a maximal invariant set contained in a proper region for system (4). The l^∞ norm of the output clearly gives no major information on the system performance during its transient. Unfortunately, in most practical cases we cannot just judge the whole system behavior through its l^∞ output norm, as this bound might prove too conservative. For example for a certain system we might accept a certain output value, provided we are sure it can be reached only for a limited amount of time. A better characterization of the transient behavior of the system, similarly to what happens when there are no uncertainties, can be obtained by checking for the presence of overshoot and, in this case, by determining its value. Since we are dealing with uncertain systems subject to exogenous disturbances, the definition of steady state output value of the step response, with respect to which the overshoot is normally evaluated, must first be introduced.

Definition 3.2 *Consider system (1) driven by an input step of the form $u(k) = U$. The steady state value of the system evolution is defined as:*

$$\mu_{ss} = \max_{d^*,w} \limsup_{t\to\infty} \|Cx(t)\|_\infty \qquad (9)$$

The definition of overshoot for the systems under consideration in this paper is then the following.

Definition 3.3 *System (1) has overshoot if there exist sequences w and d such that $\|z\|_\infty > \mu_{ss}$. In this case the positive number $\mu_{os} \doteq \mu_{inf} - \mu_{ss}$ is called the overshoot value. If $\mu_{os} = 0$ we say that the system has no overshoot.*

Note that the overshoot case is referred to the worst case: that is there is overshoot for some sequence and not necessarily for all. With these definitions we are now able to introduce the steady state and the overshoot/no-overshoot determination problem.

Problem 2 *Given an uncertain system subject to exogenous disturbances as in (1), check if the system has overshoot and in that case determine μ_{os}.*

The solution of Problem 2 is given by the next theorems and the following corollary.

Theorem 3.3 *System (1), has a steady state value $\mu_{ss} \leq \mu$ if and only the largest D^*-invariant region contained in $X_0(\mu)$ for system (4) is not empty.*

Proof. If the maximal invariant set is not empty, $\mathcal{L}$ is not empty. Since from Lemma 3.1 all the trajectories converge to $\mathcal{L}$, sufficiency follows. For the necessity note that, by definition, the limit set $\mathcal{L}$ is such that for any $\bar{x} \in \mathcal{L}$, any large $t > 0$ and any small $\epsilon > 0$, there exist w and d^* such that $\|x(t) - \bar{x}\| \leq \epsilon$. Suppose that the largest invariant set in $X_0(\mu)$ is empty, then necessarily $\mathcal{L}$ is not included in $X_0(\mu)$. Thus take $\bar{x} \in \mathcal{L}$ such that $\bar{x} \notin X_0(\mu)$. Since $X_0(\mu)$ is a closed set then there exist admissible sequences w and d^* such that the corresponding solution $x(t)$, starting from the origin, is outside $X_0(\mu)$ for arbitrary large t thus, $\mu_{ss} > \mu$. □

Theorem 3.4 *System (1), driven by an input of the form $u(k) = U$ has no overshoot if and only if for* *every $\mu \geq \mu_{ss}$ (i.e. such that system (4) admits a D^*-invariant region contained in $X_0(\mu)$), the maximal invariant set contained in $X_0(\mu)$ contains the origin.*

Proof. Suppose there exists μ such that system (4) admits a maximal invariant region P, which is contained in $X_0(\mu)$ and doesn't contain the origin. This means that the zero initial state evolution is such that $\|z\|_\infty = \tilde{\mu} > \mu$ for some sequence w and d^* (because otherwise $0 \in P$). The proof of the necessity hence follows easily by noting that μ must be equal to or greater than $\|z\|_{ss}$, i.e. the system has overshoot. The sufficiency is obvious. □

Corollary 3.1 *System (1) has overshoot if and only if there exists a value μ such that the maximal D^*-invariant set contained in $X_0(\mu)$ for system (4) is not empty and does not contain the origin. In this case, the overshoot value μ_{os} is the difference between the infimum of the values of μ for which the maximal invariant region in $X_0(\mu)$ contains the origin and the infimum value of μ for which the system admits a nonempty invariant region in $X_0(\mu)$.*

The results reported in this section can be used in conjunction with those reported in the previous section and the bisection method mentioned after procedure 3.1 to simultaneously furnish information on the l^∞ output norm of the step response of the system, the presence of the overshoot and the determination of its value, if present. Note that we have three cases: $\mu < \mu_{ss}$, $\mu_{ss} < \mu < \mu_{inf}$ and $\mu_{inf} < \mu$, apart from the critical cases $\mu = \mu_{ss}$ and $\mu = \mu_{inf}$. In the first case we have that $P^{(\infty)}$ is empty. This can checked in a finite number of steps because $P^{(\infty)}$ is the intersection of the closed sets $P^{(k)}$, ordered by inclusion. Thus $P^{(\infty)}$ is empty if and only if emptiness of $P^{(k)}$ occurs for a finite k. In the second case the condition $0 \notin P^{(k)}$ also occurs in a finite number of steps. Moreover from theorem 3.1 it follows that $P^{(\infty)} = P^{(k)}$ for some finite k. From the same theorem it follows immediately that the third case can also be checked in a finite number of steps.

Remark 3.1 *As a final remark we notice that the use of invariant regions allows us to extend these results to continuous-time systems $\dot{x}(t) = A(w(t))x(t) + Bu(t) + Ed(t)$ by introducing the Euler Approximating System (EAS):*

$$x(k+1) = [I + \tau A(w)]x(k) + \tau Bu(k) + \tau Ed(k), \quad \tau > 0. \tag{10}$$

It can be shown that as $\tau \to 0$ the maximal D^*-invariant set for this system converges to the maximal D^*-invariant set for the continuous-time system. This enables us to solve the continuous-time case by reducing it to an equivalent discrete–time problem for the EAS. Moreover, the values μ_{ss}^{cont} and μ_{inf}^{cont} for the continuous-time case are upper bounded by the values μ_{ss}^{EAS} and μ_{inf}^{EAS} computed for the EAS.

4. Example

As an example consider the second order system

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.1 & 0.7 \\ -0.7 & .55 + w(k) \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \\ &\quad + \begin{bmatrix} .3 \\ 0 \end{bmatrix} d(k) \\ y(k) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(k) \end{aligned}$$

when $u(k)$ is the unity step, $\|d(k)\|_{l^\infty} \leq 1$ and $0 \leq w(k) \leq .8$.

Using procedure 3.1 we computed the maximal invariant region contained in $X_0(\mu)$ for different values of μ and we found that the lower value for which the maximal invariant region contains the origin is $\mu_{inf}^+ = 10.86$ (the tolerance used in the bisection method is $\delta = .01$). Moreover for $\mu = \mu_{inf}^- = 10.85$ there is $\bar{k}$ such that $P^{(\bar{k})}$ is contained in $int\{X_0(\mu)\}$, which, from theorem 3.4, means that the system does not present an overshoot even if both the systems obtainable from A_1 and A_2 alone (i.e. obtained by the system above when there is no uncertainty and $w(k) = 0$ or $w(k) = .8$ respectively), present an overshoot. In fact we found the values $\mu_{inf}^1 = 1.761$, $\mu_{ss}^1 = 1.637$ for the first system and $\mu_{inf}^2 = 7.402$, $\mu_{ss}^2 = 6.831$ for the second. Finally, Figure 1 shows the maximal invariant region contained in $X_0(\mu^+)$ for the extended system (4) and the maximal invariant regions for the two systems in the absence of uncertainty when the parameter μ assumes the value μ_{ss}^i and μ_{inf}^i, $i = 1, 2$.

5. Conclusions

In most practical situations the mathematical model of a dynamic system must include some uncertainties and disturbances due to unmodeled dynamics and/or time varying conditions. This paper addresses the problem of robust performance (in the l^∞ sense) of dynamic systems subject to parametric time-varying uncertainties and in the presence of l^∞ bounded disturbances. The problem of interest is to determine a bound on the worst–case l^∞ norm of the output due to a step input, and with zero initial conditions. In principle, this problem can be recast into a standard l^1 robust performance analysis problem [10] by modelling both the disturbance and command inputs as unknown elements in the l^∞ unity ball and the model uncertainty as arbitrary LTV operators with bounded l^∞ induced norm. However, this approach will, in general, introduce a great deal of conservatism, since these results provide the worst case l^∞ bound of the output over the set of all possible l^∞ bounded inputs

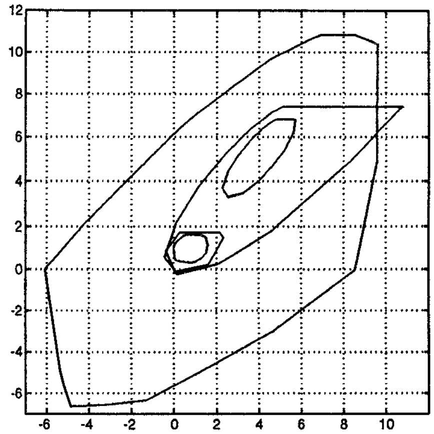

Figure 1: The maximal invariant regions contained in $X_0(\mu)$

and all possible causal LTV operators. This situation can be partially alleviated by using recent results [9] on robust l^∞ performance with mixed fixed inputs and unknown disturbances. However, at the present time these results have the form of a necessary and a sufficient conditions, separated by a non-zero gap. Moreover, they become very conservative for the case where the uncertainty is limited to memoryless time-varying gains. The main result of this paper provides a nonconservative robust performance bound for this case. This bound, obtained using a method based upon the construction of a suitable polyhedral region, can be computed in a finite number of steps.

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